Error and Misconceptions in Basic Mathematical Operations
– Third and Fourth Graders in Vanuatu –

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Abstract
The Vanuatu National Curriculum is currently enduring its reformation from an Objective base curriculum to the Outcome base approach. The previous curriculum is being substituted with the concept of the “Twin Brothers representing Vanuatu’s dual system of Francophone and Anglophone. This dual system had been harmonized to enhance pupil’s knowledge to create thinkers and problem solvers. In 2015, the new curriculum was introduced into the Education system beginning from the Grade 1 to Grade 4 levels. The other grades will progress along as the curriculum developers continued to work on them. The unique feature of this curriculum is the transition periods where pupils learned the National language (Bislama) or mother tongue at Grade 1 to Grade 3 and acquire the second language (English/French) later in Grade 4. This transition captured the interest to look into how students learning of mathematics will progress when they are transited from third to fourth grade. Thus, the purpose of this study was to discover students’ errors and misconceptions specifically in four basic mathematical operations within the transition grades.

The study was conducted in three Primary schools in Port Vila. The data were collected via diagnostic test from 204 students who were in third and fourth Grade attending these schools. The test comprised of 30 questions and 40 questions sequentially, covering the four basic operations. The findings revealed a wide range of frequent misconceptions students encountered when performing the four operations. Some of these misconceptions were, wrong place values in addition and subtraction, erroneous regrouping or no regrouping in addition and subtraction, addition error in multiplication, wrong multiplication and division expression and so forth. The study visualized these misconceptions as the evidence indicating the lack of comprehending fundamental concepts of basic operations, as well as the techniques in calculation of operations and their relationships and procedures among the transition graders.

1. Introduction
Errors and misconceptions in mathematical operations demanded deeper exploration for the advancement of mathematics education in Vanuatu. Teachers must acquire skills and have a deeper knowledge in Mathematics in order for their students to pursue in mathematics. They need to be an expert in choosing from a variety of pedagogical and assessment strategies that will eliminate students’ misconceptions in basic operations. They need to know the ideas which students often have difficulty and ways to help bridge common misunderstandings among the transition graders.

When students possess false knowledge, believed to be true knowledge, it may possibly mislead their perception of mathematics. To advance teachers’ mathematics understanding and improve instructional pedagogy, mathematic educators must find effective ways to deal with students misconceptions. This study discovers mathematical misconceptions that are affecting students learning of the four basic operations within the transition graders of elementary schools in Vanuatu. It also highlighted the associated knowledge which may lead to the growth of misconceptions in learners. The study also suggests
some recommendations for further discoveries on misconceptions in mathematical operations for the benefit of mathematics education in Vanuatu.

2. Literature Review

As part of our human culture and heritage, mathematics is one of the core subjects for developing essential life skills in individuals. The mathematical knowledge and skills have great impact on both everyday life and professional activity. It provides foundational knowledge and skills for other school subjects such as science, art, economy, technology and engineering (Bessot, 1996). Mathematics also produces more citizens who can learn and think creatively, critically and logically, no matter their career fields (Mihriban, 2017; Moreira, 2012; Michael et al., 2011). Bobby (2015) point out that achievement in mathematics education determine the success and progress of the nation, both in improving the quality of education and in political participation. Mathematics education develops the growing sense of confidence and a sense of ownership for the future as agents of change (Keeno, 2017). Keeno (2017) further explained that in students, mathematics education contains values such as transforming difficulties into challenges as a form of self-satisfaction, motivating and stimulating appreciation of the success of students, developing self-confidence, and reasoning. Serving this reason, all students should be well educated in mathematics in order to be competitive in the 21st century global economy (Bessot, 1996).

3. Research Goal

The overarching goal of this study was to determine the underlying misconceptions among the transition graders in Vanuatu elementary schools when solving mathematical operations with basic addition, subtraction, multiplication and division.

The main objective was to examined the consistency of students’ error patterns between the addition, subtraction, multiplication and division and investigate how these errors and misconceptions influenced students’ performance on solving mathematical operations when they are transited in terms of language.

The research questions that were discovered were:

» What are possible errors and misconceptions in teaching and learning basic mathematical operations in mid elementary schools in Vanuatu?

» What are the causes of errors and misconceptions in teaching and learning basic mathematical operations in mid elementary schools in Vanuatu?

» What kind of remedies teachers would take to treat errors and misconceptions in teaching and learning basic mathematical operations in mid elementary schools in Vanuatu?

4. Methodology

The targeted population of this study was the population of mid elementary students attending English speaking schools in Vanuatu. The targeted grades were third and fourth graders.

Of the study population, the initial sample was a random selection of three schools. The schools were government and governments assisted schools. Two of these schools were located in urban and one in the rural. The total participated students were 204 third and fourth graders. The third and fourth graders were selected as they are the two grades where the transition within the new curriculum befalls.

It was plausible that few limitations could have influenced the results obtained in this study. Firstly, it may be too early to investigate students’ error and misconceptions in long division especially with third graders. The study was undertaken in term two of the academic year where students did not yet cover much as expected from the test. Secondly, not all students attending these grades were present during the test. Finally, the test was given only to students attending English speaking schools while the issue is also affecting both English and French speaking students.

Data were collected through students’ responses in a diagnostic test of 30 questions in third grade and 40 questions in fourth grade. The test was composed of five sections. There were sections of addition, subtraction, multiplication, division and word problems. Students explain their response to each question through vertical operations using the space provided on the test paper.
5.1. Data analysis

Data collected were analysed based on the accuracy rate of each question and the proportion of error rate detected for particular misconception. The data collected from each grade were recorded separately first then later combined when categorizing and coding the errors and misconceptions detected. These results were put together to determine the kind of errors and misconceptions students frequently tend to have in the four basic operations. The discussions below highlighted the frequent misconceptions detected in each section of the test.

5.2. Errors and misconceptions in addition

This study discovered four frequent misconception students encountered when resolving addition problems of different conditions. These misconceptions were 'wrong place value', 'wrong operations', 'no regrouping' and 'wrong regrouping'. The 'wrong place value' was defined as the misplacement of digits in place value when performing addition. The 'wrong operation' here was characterized as left to right operations. The 'no regrouping' was categorised as failure of carrying over when a sum of a place value resulted in two figures. The 'wrong regrouping' was categorized as inattention to carry over digits. This is when students forget to add after carrying over. The 'wrong place value' misconception is highlighted below as evidence to students' misconceptions in addition as indicated above.

Twelve questions with different conditions of addition were identified having significant results baring 'the wrong place value'. One of the conditions of addition that portrayed this misconception was the responses in Q.2 of both grades. The purpose of these questions was to explore students understanding of place value in addition of one digit addends against two digit addends with regroupings. The results baring the 'wrong place value' in these questions are presented in Table 1.

The results in Table 1 explained that 'wrong place value' here resulted from students' lack of the correct visualization of break ten and make ten. The third grader in the sample thought of 8 in the second addend as tens as he/she adds with 4 in the place of tens in the first addend to get 12. This child was not able to see the meaning of 8 as how many more does it needed to make ten. Here 8 needed 2 more to make ten. Which meant 8 was representing the ones place since it needed 2 more in order to represent tens. And by bringing down the 7 in the ones place leaved the child with the incorrect concluding sum of 127. The child in fourth grade also encountered similar errors.

These findings revealed that students encountered 'wrong place value' in addition when the quantities of digits in addends were not equal. Students' responses unveiled that 'wrong place value' in addition may possibly resulted from the absence of the practical and theoretical understanding of the decimal notation system in students. Students lack the understanding of base ten. In fact most students performed the operation from left to right showing how their associated the concept of left to right reading and writing procedures with the concept of addition.

5.3 Error and misconceptions in subtraction

The findings through this study discovered three frequent misconception students encountered when executing subtraction of different conditions. These misconceptions were 'wrong place value', 'no regrouping' and 'wrong subtraction'. The 'wrong place value' in subtraction was classified here as the misplacement of digits in place value when performing vertical subtraction. The 'no regrouping' was categorised as failure to borrow when a minuend was less than the subtrahend in a place value. The
‘wrong subtraction’ was classified as the process of reversing the subtraction procedures in place values. The ‘no regrouping’ is explained below as evidence of the three frequent misconceptions.

There were twelve questions from the study detected as evidence holding significant results prior to ‘no regrouping’. The findings in Q.15 in third grade and Q.19 in fourth grade unveil the discoveries of ‘no regrouping’. These two questions were expected to investigate students understanding of place value and regrouping in subtraction of two digit subtrahends from three digit minuend. The evidence of ‘no regrouping’ in these questions are portrayed in Table 2.

According to the results in Table 2, the samples presented for this investigation showed no representation of regrouping in both graders. Prior to the size of subtrahends and minuend in the ones and tens place, both students determined to reverse the procedure of subtraction. Since the minuends in the ones and tens place were less, the third grader began by subtracting 3 from 8 to get 5 in the ones place. Here the child was not able to define 3 and 8 before performing the operation. The child would have only identify the difference and perform the borrowing if he/she visualize 3 as 7 more to make ten and 8 as 3 more to make ten in order to comprehend that 3 is less than 8 and that it needs one more tens in order to give away 8. In the same way, the child subtracted 4 from 5 to get 1 in tens place and brought down 8 in hundreds place to get an ending results as 815. Similar incorrect operation is demonstrated in fourth grade sample.

These results explained that students encountered ‘no regrouping’ when performing subtraction that involved borrowing. The study assumed that this misconception may base on students understanding of subtraction as taking away only. Believing that in any situation, only when a digit is greater there is possibility to give away. The misconception arose when students were comparing digits in single place value. They were not able to comprehend subtraction as findings the difference. They were as well not able to define the correct meaning of each digit. The reason here was rooted in students' lack of visualizing of make ten and break ten.

5.4 Errors and misconceptions in Multiplication

The findings through this study discovered four possible misconceptions students frequently encountered when solving multiplication problems. These misconceptions include ‘wrong multiplication expression’, ‘wrong operation’, ‘no carrying over’ and ‘addition error’. The ‘wrong multiplication expression’ here was defined as the misplacement of multiplicand and multiplier. The ‘wrong operation’ was categorized as left to right operation as well as operating as subtraction or addition. The ‘no carrying over’ was underlined as failure to carry over when a product of single place value was delivered in two digits. And the addition error’ was classified as incorrect carrying over, and ignorance of adding after carrying. The misconception of ‘no carrying over’ is highlighted below as evidence of the four misconception detected.

Four questions with significant outcome were detected in fourth grade possessing this misconception. The findings through these questions revealed that a category of students who participated in this study were experiencing this misconception when performing multiplication that involved addition and carrying over. One of the questions highlighting these results was Q.29 in fourth grade. The aim of this question was to explore students understanding of multiplication of three digit multiplicand against one
5.5 Error and misconceptions in division

The findings through this study discovered ‘wrong division expression’ and ‘wrong operation’ as the most frequent misconception in division of any condition. The ‘wrong division expression’ was defined as the misplacement of dividend, divisor and quotient in long division. The ‘wrong operation’ was categorized as operating as addition or subtraction. The study also discovered other misconceptions like multiplication error, subtraction error, bringing down error, and remainder error as very influential as well. However, they may need further discoveries since students did not cover much on division beforehand in order to provide as much necessary information concerning the influence each one had in students’ learning of division. Further study on such theme may be conducted again but rather in term three of an academic year. The findings prior to ‘wrong division expression’ are highlighted below as evidence of the two frequent misconception detected.

There were eight questions from the study bearing significant results prior to this misconception. The findings through these questions revealed that most students participating in this study encountered this misconception when performing long division of any condition. The findings through Q.24 in third grade and Q.33 in fourth grade conferred the discoveries.

<table>
<thead>
<tr>
<th>Table 3: The ‘No Carrying Over’ in multiplication</th>
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<tr>
<td><strong>Gr. 4 Q.29</strong></td>
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<tr>
<td><strong>Sample 1</strong></td>
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<tr>
<td><strong>Accuracy rate</strong></td>
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<tr>
<td><strong>Proportion of error rate for ‘no carrying over’</strong></td>
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<tr>
<td><strong>Error sample</strong></td>
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<td><img src="image1.png" alt="Error sample image" /></td>
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*Source: Made by Aurther based on data analysis*

digit multiplier. Additionally, the question was also purposed to explore students’ knowledge of addition and carrying over in multiplication. The findings of ‘no carrying over’ in this question are presented in Table 3.

Results from Table 3 justified that students suffered ‘no carrying over’ in various conditions. The condition from the first child portrayed the failure to carry over in the product of tens and the ones place. The second child indicated ‘no carrying over’ in the ones place only. In these operations, the first child began by multiplying 5 and 8 in the ones place to get the product 40. The child wrote 40 as the product ignoring to carry over the 4. In the same way, the child multiplied 8 and 8 and get 64 as the product in the tens place. Both products showed the child ignoring to carry over leaving the incorrect concluding result as 6440.

These findings revealed that students suffered ‘no carrying over’ when solving multiplication problems that involved addition and carrying over. The results underlined that this misunderstanding resulted from students’ lack of the techniques to calculate multiplication as well as the lack of the concept of multiplication as continues addition.

<table>
<thead>
<tr>
<th>Table 4: The ‘Wrong Division Sentences’</th>
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<tr>
<td><strong>Gr. 3</strong></td>
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<tr>
<td><strong>Q.24</strong></td>
</tr>
<tr>
<td><strong>Accuracy rate</strong></td>
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<tr>
<td><strong>Proportion of error rate</strong></td>
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<td><strong>Error sample</strong></td>
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*Source: Made by Aurther based on data analysis*
These questions were aimed to explore students understanding of long division with one divisor against two dividends. Additionally Q.32 was also aimed to explore students understanding of remainder in division. The findings of ‘wrong division expression’ in these questions are presented in Table 4.

The results in Table 4 explained that students did not acquire the meaning of division as distributing equally the number by groups. Students were not able to visualize the meaning of dividend, divisor and quotient. None of them indicated the correct placement of dividend, divisor as well as the quotient.

They wrongly interpreted division as addition, subtraction or multiplication. For instance, regardless the incorrect placement of digits, the child in Q.24 got an incorrect ending result of 16 by adding the 12 and 4. This response as well as others indicated that students lack the techniques to calculate long division.

5.6 Error and misconceptions in word problem

The findings through this study discovered ‘wrong interpretation’ as the common misconception students encountered in word problem. This misconception was classified as incorrect interpretation of problem situation. All the nine questions provided in this section of the test detected significant results holding this misconception. Q.28 in third grade and Q.37 in fourth grade justify this misconception. These questions were aimed to investigate students understanding of subtraction in word problem. Table 5 reported the findings of ‘wrong interpretation’ in these questions.

According to the results in Table 5, the samples represented gave details of the cause of this misconception. One of the causes was students’ understanding of sentences. When observing the responses to these questions there was an indication that students struggled to understand the sentences provided for each problem. It was why some students performed addition problems as subtraction, and multiplication problems as addition and so forth. They might as well do not understand the mathematical terms used as index to the solution of the problem. This is the reason both students above added instead of subtracting. For instance, the mathematical term ‘left’ or remaining was provided as guide in these questions. However, regardless the incorrect place value and the computation error in sample Q.28, both samples showed students performing the operation as addition instead. The child in Q.28 either begun by adding 2 and 1 to get 3 in the hundreds place. The sum 14 in the tens place showed the child adding 6 and 8, and by bringing down the 4 in the ones place gave the child an incorrect ending result as 3144. These incorrect responses indicated that students did not acquire yet the techniques of interpreting word problems. There was no indication of how important information/data/number in the problems was to be collected. There was no visualization of tables/diagrams/graphs from the students showing their understanding of the problem. The incorrect math expression indicated wrong interpretation and wrong solution to the problems.

6. Conclusion

The findings through this diagnostic study discovered that students participated in the study had quite shallow understanding and skills in all four basic operations as highlighted through frequent misconceptions detected. These findings prior to the frequent misconception revealed the possibilities that third and fourth graders in Vanuatu will possibly encounter these errors and misconceptions when performing basic mathematical operations. Thus, understanding of students’ conceptions, ideas, and
skills of mathematical operations is central to good teaching and learning of mathematical operation. It is necessary for elementary teachers to understand the process of treating students’ errors and misconceptions well in order to effectively facilitate mathematical concepts accurately with elementary learners within the transition graders.

7. Recommendations

This study was considered as a channel that leads to future research on basic mathematical operations in Vanuatu. As far as this is concern, there are few recommendations that need to be considered. Firstly, it is recommended that an additional research on division must be carried out to discover more on the detected misconceptions highlighted. Secondly, it is recommended that a deeper study on students’ misconception must be carried out to prove students’ misconception as an element of treating students’ development of mathematical concepts within the transition graders in Vanuatu. Additionally, this study recommends for specific study on individual operation rather than combining four operations to allow a researcher to explore sufficient possibilities of addressing each misconception.

References