Study Note

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An Analysis of 5th Grade Fraction Magnitude Comparison Tested to 7th Grade Primary School Students in Papua New Guinea (PNG)

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Abstract: Fractions concept poses a challenge for many primary school teachers and students all over the world though it is important for future mathematics learning. Many recent research findings have uncovered that both teachers and students in Papua New Guinea at primary school level have difficulty understanding the fraction concept.

This report is based on a sample 5th grade fraction magnitude comparison test that was administered to the 7th grade students of one of the primary schools in Papua New Guinea (PNG). The purpose of this study is to comprehend the effectiveness of the mathematics curriculum at primary school level in PNG. A test consisted of 6 fractions comparisons items were administered to a total of 76 seventh grade students from two classes. The study engaged both quantitative and qualitative methods of data collection. The findings exposed students' limited understanding of fraction magnitude and common areas of misconceptions. Hence, the study proposes more effective ways to improve the standards of mathematics education at the primary school level in PNG.

Keywords: Papua New Guinea, Fraction magnitude, Misconceptions

1. Introduction as Study Background

Papua New Guinea has four levels of education; namely, elementary, primary, secondary and tertiary. The basic or compulsory education is made up of 3 years elementary and 6 years primary from the existing structure of 3-6-4 (3 years Elementary School-6 years primary school-4 years secondary school). Mathematics subject is compulsory for all levels of PNG education starting from the elementary level.

This report contains information about sample mathematics test conducted in one of the primary schools in Nations Capital, Port Moresby, and PNG. The study is part of the program sponsored by JICA under the long term study program "*Improvement of Quality of Teaching Materials for Mathematics and Science*". Hence, the sample acts as tool to guide curriculum planners and educators about the general misconceptions of teaching and learning in mathematics education as well as the effectiveness of the existing curriculum so that applicable measures can be taken to improve the standard of mathematics education at the primary school level in PNG.

Port Moresby is the biggest city and the economic centre of the country; therefore the students and teachers were from most provinces of PNG. Also the classes consisted of children from various socio-demographic backgrounds with wide range of ability levels.

The primary schools in PNG can be classified as rural, semi-urban and urban schools. Possibly it is right to say that this sample represented the urban centre of the country, and there may well be different results in the semi-urban or rural schools of Papua New Guinea. However, previous study (Apule, Ishizaka, Osawa, & Kosai, 2016) discovered that the urban school performance level was just equal as the performance level of rural and semi-urban schools. Hence, this sample study can signify efficiency of primary school level mathematics education in PNG.

2. Participants

The sample includes two seventh-graders (ages 12-15) in a primary school in Port Moresby, the National Capital of Papua New Guinea. The sample of 76 students from two classes (38 male and 38 females) participated in this survey.

3. Instruments

The main source of data collection was through the six (6) open-response fraction items used in the test purposely to gather information about students thinking and understanding of fractional concepts. The test required students to use the inequality symbols (>, = or <) to compare the fractions and explain their answers using words and or diagrams on the space provided. Thus the study engaged both the quantitative and qualitative methods of data collection. The open-ended aspect of the test was to provide a qualitative data of students understanding of fractions concepts and expose their tendencies in thought.

Table 1: Fraction pairs for comparison and their type used in the test.

Item	Pre-test	Fraction Comparison type
a.	$\frac{5}{6}$, $\frac{2}{6}$	Same denominators
b.	$\frac{1}{3}$, $\frac{1}{2}$	Unit fractions/same numerators
c.	$\frac{1}{2}$, $\frac{2}{4}$	Equivalent fractions
d.	$\frac{5}{6}$, $\frac{3}{4}$	Different Denominators
e.	$\frac{2}{3}$, $\frac{3}{5}$	Different Denominators
f.	$\frac{3}{5}$, $\frac{3}{4}$	Same Numerators/different denominators

4. Results

4.1. Overall results

The graph (figure 4.1) shows the overall results of the sample test performance.

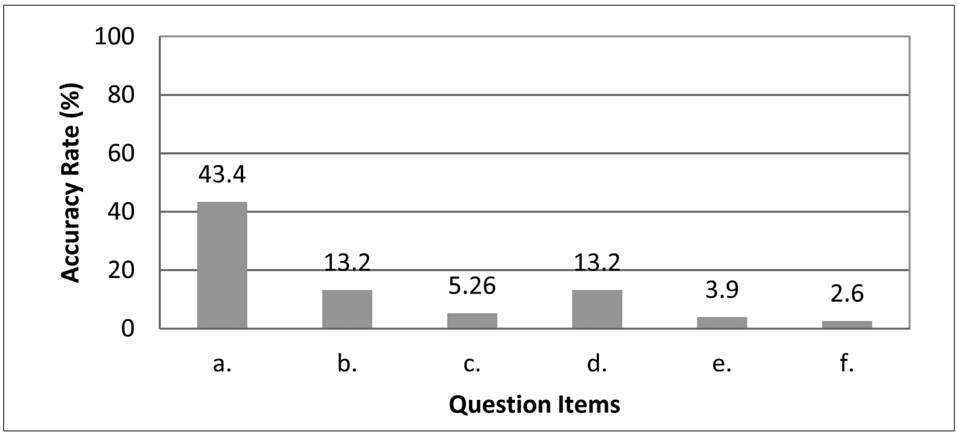


Figure 4.1: Overall performance

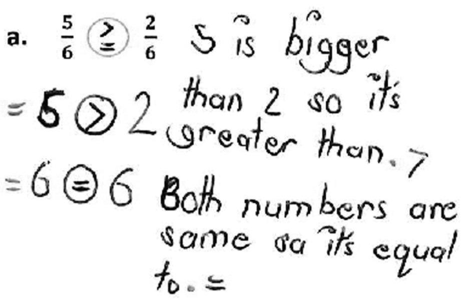
According to the overall performance in the test, less than half (43.4%) of the total respondents had correct answers for item ‘a’-comparison of fraction with the same denominator. Otherwise, the other five comparison items were poorly performed by the total respondents, exposing similar type of misconception through their words and or picture diagrams used in explaining their comparison (>, =, <) choices. For item ‘b’ comparison of unit fractions ($\frac{1}{3}$,$\frac{1}{2}$) the accuracy rate was 13.2% while item ‘c’, comparison of equivalent fraction ($\frac{1}{2}$,$\frac{2}{4}$) was only 5.26%. The two other items with different denominators items ‘e’ and ‘f’ was the worst performed. Three students (3.9%) had correct response for item ‘e’ whereas only two students (2.6%) had item ‘f’ correct.

4.2 Qualitative Analysis

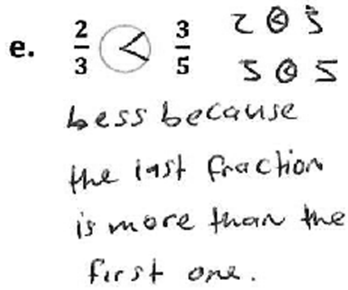
As per the research design, students were encouraged to write using words or picture diagrams to justify their comparison choice (>, =, <) for each pair of fraction comparison. These data type was analyzed qualitatively by using an open coding to code each student case independently. These independent students’ cases were then combined through checking the commonalities and strategies used. Subsequently, four (4) main categories were identified in which all the independent codes were merged. Three of these categories were as an evidence of students' misconceptions while the fourth category was as an evidence of correct reasoning for the fraction magnitude comparisons.

(i) Misconception 1(M1): Viewing a fraction as two separate whole numbers

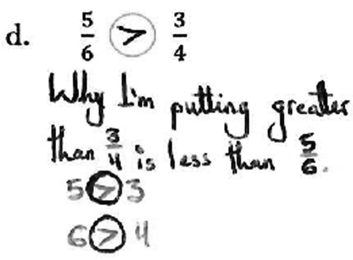
Misconception 1 (M1) was due to students viewing a fraction as two separate whole numbers. The sample students' responses were used as an evidence of M1.



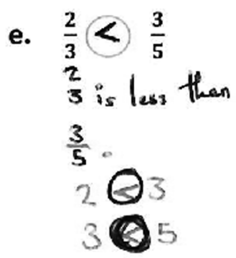
5 is bigger than 2 so its greater than >, 5 >2, Both numbers are same so its equal to 6 = 6



2<5, 3<5, less because the last fraction is more than the first.



Why I'm putting greater than ¾ is less than 5/6, 5>3, 6>4



2/3 is less than 3/5, 2<5, 3<5

Figure 4.2: Sample evidences of Misconception 1 (M1)

The sample evidence was used to compute the rate of M1 for the sample respondents. Figure 4.3 shows the rate of Misconception (M1) observed in each item.

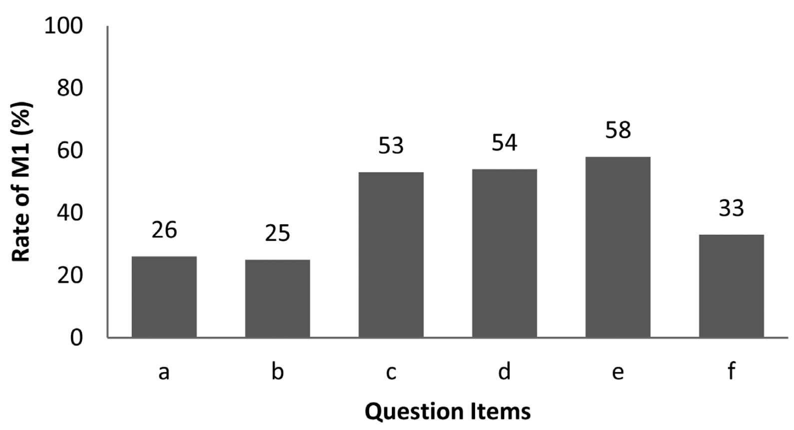


Figure 4.3: Rate of Misconception 1 (M1).

According to the information provided in figure 4.3, we can see that M1 was noticed in all items with the highest of 58% for items 'e', followed by item 'd' with 54% and 'e' with the M1 rate of 53%. The three other fraction comparison question items had the following M1 rates, item 'a' (26%), item 'b' (25%) and item 'f' (33%).

Correspondingly, it was observed that many students reiterated M1 for all the question items. For example, same denominator fraction item 'a' ($\frac{5}{6}, \frac{2}{6}$), students compared the numerator and denominator separately, i.e. 5>2 (5 is greater than 2) and 6=6 (Both numbers are same so they are equal) thus their comparison choice was ($\frac{5}{6} > \frac{2}{6}$). Though these students' answers were correct, their thinking was wrong because students with this misconception applied the same strategies to all the other pair of fraction comparison items, like item 'b' ($\frac{1}{3}, \frac{1}{2}$), 1=1 and 3>2, so their answer was ($\frac{1}{3} > \frac{1}{2}$). Since the numerators were same, they just compared the denominators.

Similarly, for item 'd' students with category (i) misconception claim that $\frac{5}{6}$ was greater than $\frac{3}{4}$ because 5>3 (comparing the numerators separately) and 6>4 (comparing the denominators separately) and thus $\frac{5}{6}$ has bigger numerator and denominator so it is bigger than $\frac{3}{4}$. Though their answer was correct, students with this misconception applied the same strategy to compare item "e", claiming that $\frac{2}{3}$ is less than $\frac{3}{5}$ because 2<3 (i.e. comparing the numerators separately) and 3<5 (i.e. comparing the denominators separately) because the fraction number $\frac{3}{5}$ has bigger numbers for numerator and denominator than fraction number $\frac{2}{3}$, which is a total misconception. These two items were deliberately prepared to capture this type of misconceptions.

(ii) Misconception 2 (M2): Viewing that same numerator fractions are equivalent

The second Misconception (M2) was identified to be students viewing that same numerator fractions are equivalent. The sample students' responses were used as an evidence of M2 as shown in figure 4.4 below.

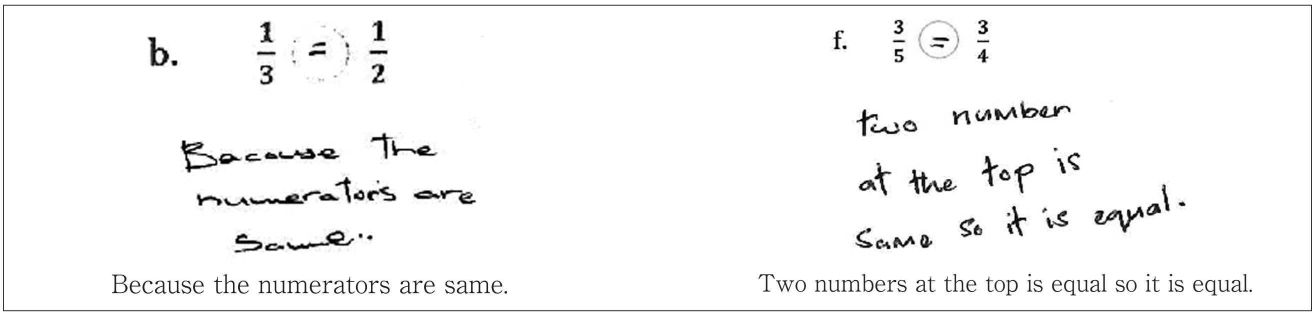


Figure 4.4: Sample evidences of M2

The sample evidence was used to compute the rate of M2 for the sample respondents. Figure 4.5 shows the rate of M2 in the sample test.

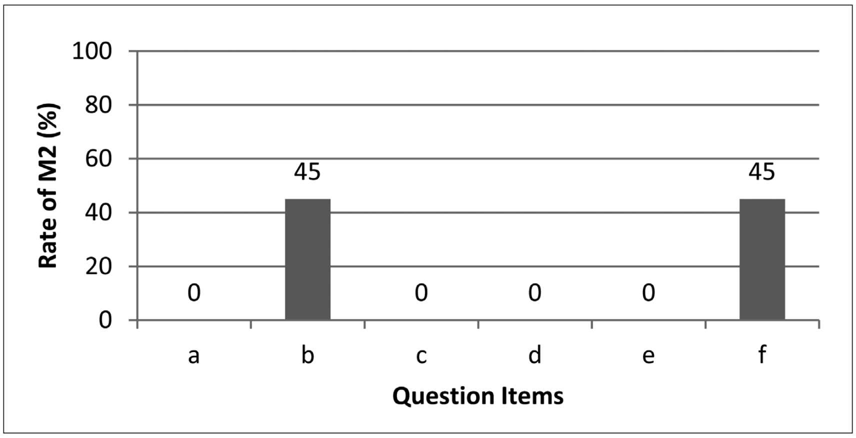
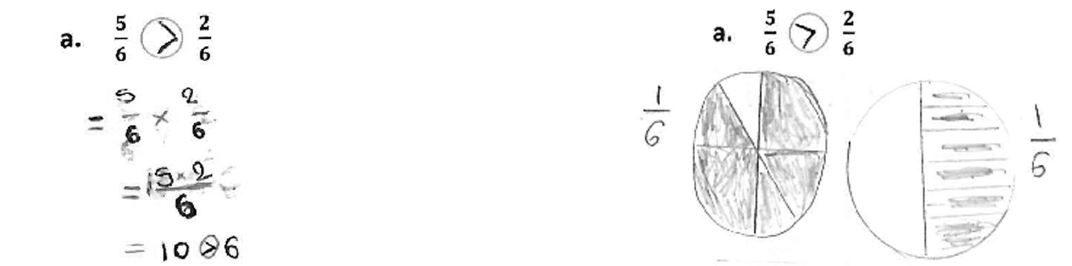


Figure 4.5: Rate of Misconception 2 (M2).

As it can be seen from the graph in figure 4.5, M2 was only noticed in question items 'b' ($\frac{1}{3},\frac{1}{2}$) and 'f' ($\frac{3}{5},\frac{3}{4}$) because these two items have same number as numerator for each fraction pair comparison. Each item had M2 rate of 45% because the same group of students who had this misconception view in item 'b' reiterated the misconception in item 'f'. These students decided to focus on comparing the numerators of the two fractions rather than thinking about the fractional amounts.

(iii) Misconception 3 (M3): Incorrect reasoning for fraction magnitude comparison

The third Misconception (M3) was identified as "incorrect reasoning" basing on students responses. The M3 includes use of; addition, multiplication, or incorrect diagram and or picture representations. Figure 4.6 shows the sample students' responses as an evidence of M3.



5/6×2/6 = 5×2/6 = 10>6

Incorrect diagram representations

Figure 4.6: Sample evidences of M3

The sample evidence was used to compute the rate of M3 for each item. These types of misconception were noticed in all question items. Figure 4.7 shows the distribution of M3 in each item.

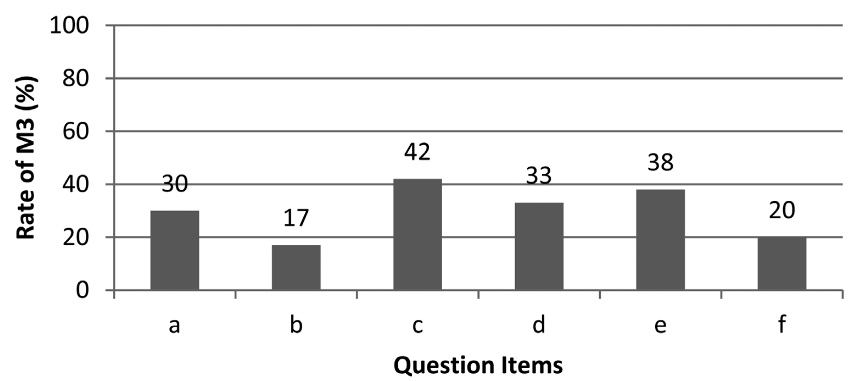


Figure 4.7: Rate of Misconception 3 (M3)

According to figure 4.7 above, very high rate of M3 was noticed in all the fraction magnitude comparison pairs of item. The rate of M3 ranges from lowest as 20% (item 'b') to the highest as 42% (items 'c'). In addition, it can be seen that equivalent fractions item "c" ($\frac{1}{2},\frac{2}{4}$) recorded the highest rate of M3 because the sample respondents could not reason the equivalent nature of the fractions or could not view these fractions as equivalent due to use of operations such as addition, multiplication or incorrect picture diagram representations.

Hereafter, the high rate of M3 as observed in these results perhaps reveal that students were not taught fraction magnitude comparison topic or had missed out on learning this very important mathematic topic.

5. Discussions and Implication

The sample population overall performance was very poor exposing limited understanding of fraction magnitude in all the six pairs of fraction comparison items. The most straightforward, comparison of same denominator fractions question item 'a' ($\frac{5}{6}, \frac{2}{6}$) had an accuracy rate of 43.4%. On the other hand, five other pair of fraction comparison items having different denominators proved to be very difficult for PNG primary students at this grade level. The worst performance score was for item 'e' ($\frac{2}{3}, \frac{3}{5}$) and 'f' ($\frac{3}{5}, \frac{3}{4}$) with the accuracy rate of only 3.9% and 2.6% respectively. Similarly, equivalent fractions item 'c' ($\frac{1}{2}, \frac{2}{4}$) recorded an accuracy rate of 5.2% from the total respondents'. The students' respondents could recognize the equivalence nature of the two fractional numbers. Likewise, the comparison of unit fraction item 'b' ($\frac{1}{3}, \frac{1}{2}$) had an accuracy rate 13.2% while different denominator fraction item 'd' ($\frac{5}{6}, \frac{3}{4}$) also had an accuracy rate of 13.2%.

The further qualitative analysis identified three types of common misconceptions for each type of fraction magnitude comparison items in this study. The most common misconceptions was "*viewing a fraction as two separate whole numbers*", mostly treating the numerators and denominators separately. Students think that the numerator and denominator are separate values and have difficulties seeing them as a single value. For example, fraction comparison question item 'e' ($\frac{5}{6}, \frac{3}{5}$), students simply compared numerator to numerator 5>3, and denominator to denominator 6>5, concluding that $\frac{5}{6} > \frac{3}{5}$,or because the first fraction has bigger numbers for its numerator and denominator over the second fraction. Though the answer is correct, their thinking is incorrect. It is hard for them to see $\frac{5}{6}$ and $\frac{3}{5}$ as individual numbers. Many students with this view of misconception were noticed to have applied the same reasoning to all the questions, totally reducing the success rate.

For comparison of unit or same numerator fractions such as question items 'b' ($\frac{1}{3}, \frac{1}{2}$), and 'f' ($\frac{3}{5}, \frac{3}{4}$) prompted Misconception 2 (M2) of viewing that the same numerator fractions are equivalent. The sample respondents recorded M2 rate of 45% for each questions items 'b' and 'f'. For example for the fraction comparison pair $\frac{1}{3}$ and $\frac{1}{2}$, students with M2 say that these two fractions are equal because the numerators are equal, 1=1. It was observed that mostly the same group of students having this view in item 'b' repeated the same view in item 'f' by focusing on comparing the numerators of the two fractions rather than thinking about the fractions amounts.

Furthermore, in all the questions items, students exposed total confusion state of having the skill of comparing fraction magnitude by applying unnecessary operations such as addition or multiplications, or even incorrect pictorial representations. These responses were categorized as Misconception 3 (M3) due to incorrect reasoning for fraction magnitude comparison. These misconceptions were noticed in all the question items with lowest as 17% in item 'b' and highest as 42% in item 'c'. These students simply disclosed inadequate understanding of fraction magnitude.

6. General Conclusions

The primary purpose of this study was to identify the effectiveness of PNG primary school level mathematics curriculum so that appropriate measures can be taken to address the findings highlighted in the study. The study was successfully carried out in one the primary schools in Port Moresby, PNG. There was a fair (50% male and 50% female) participant from both genders from the 76 total participants. The sample mathematics test consisted of items taken from 5th grade Japanese Curriculum on the content area of fractions, specifically comparison of fractions magnitude.

The results of the test administered showed that majority of the students fell short of demonstrating mastery of the 5th grade fraction comparison concepts at the 7th grade primary school level in PNG. Students exposed very limited understanding of fraction magnitude and its comparison skills. The qualitative analysis exposed three common areas of misconceptions in comparing fractions magnitude. Misconception 1 was due to students treating the numerator and denominator as separate whole numbers. Misconception 2 was due to students viewing that same numerator fractions are always equivalent, and Misconception 3 was identified as incorrect reasoning's, that includes the use of operations such as addition and multiplications and incorrect diagram representations.

Hereafter, in light of the findings revealed in this study, the following recommendations need careful attention in order to raise the standards of mathematics at the primary school level in PNG.

Firstly, Teaching and learning of fractions with understanding requires significant effort from both teachers' and students. Teachers must help students to recognize that fractions are numbers and that they expand the number systems beyond whole numbers. Use number lines as a central representations tool in teaching this and other fraction concepts from the early grades. It is also important to incorporate variety of manipulative such as set models, area models, number lines etc…This will help students to explore fractions with a variety models and connect the visuals to the related concepts. For example, an area model helps students to visualize parts of the whole. A number line shows there is always another fraction to be found between any two numbers- an important concept that is underemphasized in teaching of fraction. Using number lines are very important in developing students understanding of the fraction yet they are not widely used in PNG classrooms. Recent reviews of research on fraction (Siegler et al, 2010) report that the number line helps students understand a fraction as number (rather than one number over another number) and develop other fraction concepts.

Secondly, the standard of Papua New Guinea's education system would not be elevated until the country begins to produce a greater number of qualified teachers. It was observed that generalized subject teachers were teaching the upper primary classes, where one teacher teaching seven or eight different subjects though not competent with some of these subjects which compromises the quality of education students deserve. It is important that specialist subject teachers' must be assigned to important subject like mathematics so that the intended curriculum is properly implemented to raise the standards of mathematics educations. Also it is strongly recommended that teachers must be provided with adequate in-service training on mathematics content in line with applicable materials and methods that can support students learning in the classroom.

Finally, there is a possibility that many PNG primary school teachers skip or overlook some of the important mathematics topics and subtopics in the existing curriculum. Students showed little or no understanding of fraction magnitude comparison skills in this study. Hence, curriculum alignment for primary school level must be clearly stated and spelt out to the teachers' so that appropriate content at each grade is delivered to the students. That is the primary school level curriculum must build new ideas and skills on earlier ones within lessons, from lesson to lesson, from unit to unit and from year to year while avoiding excessive repetition. As students construct and develop new ideas and skills, the concepts and processes they learn become richer and much more complex.

To conclude, fractions arithmetic is fundamental for future mathematics achievement and for ability to succeed in many professions. Unfortunately, these skills bear large difficulties for many teachers' and students. Therefore, it is important to review and identify why learning fractions arithmetic is so difficult for primary school level in PNG. It is better to identify the current practices and commonalities and see what interventions are effective that can help children overcome the challenges of mastering fraction arithmetic.

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