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On the Significance of Alternative Mathematics in a School context

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1. Introduction

In epistemology, we can take various positions on the question, "What is mathematics?" If we stand on absolutism, mathematics will be regarded as a truth that leaves human minds and is a static and immutable fact. On the other hand, if we stand on relativism (especially on constructivism), mathematics will be the expression of human minds and viewed as an object constituted for the purpose of aiming at various value realizations (c.f., Hersh 1997, Ernest 1998). The problem of how students approach to mathematics has a fundamental influence on the teaching and learning of mathematics (Dossey, 1992). So, how do Japanese students and teachers view mathematics and mathematics learning? According to the Second International Mathematics Study (SIMS) by IEA, Japanese junior high school and high school students respond negatively to questions such as "mathematics will change quickly in the future" or "mathematics is suitable for people who are going to consider new concepts by themselves" (NIER, 1991). It appears that Japanese students regard mathematics as an inflexible, finite and closed subject. In addition, according to the Third International Mathematics and Science Study (TIMSS), it was shown that the response of about 60% of Japanese mathematics teachers to the question of what was required for students who do mathematics was "memorizing formulas and procedures," which is far above the international average of 39% (NIER, 1997). These tendencies cannot be regarded as positive results for the following two reasons : Firstly, it is hard to get students to get involved in enthusiastic learning and positive participation if mathematics is viewed as a complete and absolute system (Minato, Hamada, 1994). School education has the role of raising mathematical culture and education from generation to generation. At the same time, it is teachers must open up in the minds of students the possibility of new forms of mathematics by indicating that current mathematics is only the realization of the intellectual activity of our predecessors. Secondly, regarding mathematics as an absolute thing results lost opportunities for intellectual tolerance in mathematics education. Our basic stance is one where individuality is respected. This stance leads to intellectual tolerance that is not bound by one certain standard, but accepts differing views and where people learn from each other. We considered it as the realization of multicultural education or ethnomathematics (cf. Bishop, 1995, Gerdes, 1996). Here, we accept the necessity of improving mathematics education, and allow for relativism and regard mathematics as a fallible system that is always open to development or correction. When premised on the relativity and fallibility of knowledge, the objectivity of mathematical knowledge can be no more than an open subjectively shared concept in a certain community (Vergnaud, 1987). With this meaning, the mathematical knowledge we acquire is no more than provisional knowledge that is then investigated on the basis of relative stability, rather than certainty. If we accept the relativity and/or fallibility of mathematics, the existence of "alternative mathematics" (Bloor, 1976/1985) will inevitably be accepted and be seen as something that is different from general mathematics or school mathematics. The educational implications need to be considered.

The purposes of this paper are : 1) to determine the significance of "alternative mathematics" in a school context, and 2) to point out the problems of

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applying it. For this purpose, we advanced in the following manner: First, the social applications of mathematics and alternative forms of construction are shown through an outline of the thoughts of David Bloor, who raised the idea of "alternative mathematics" (Chapter 2). Next, after submitting various viewpoints, we extend the range of the argument on "alternative mathematics" to school education (Chapter 3). Finally, we cover the possibilities of improving the attitudes of students and realize the aim of school mathematics by constituting "alternative mathematics."

2. Relativity of Mathematical Knowledge and "Alternative Mathematics"

David Bloor, a philosopher of science at Edinburgh University, reflects on the history of science from a sociological point of view. He has analyzed the variation in mathematics that derives from social factors. Bloor has pointed out that theory and method in mathematics are just an agreement (convention) defined socially to the last, and do not have elevated claims to absoluteness or objectivity (Bloor, 1983). Since the appearance of non-Euclidean geometry by Bolyai and Lobatchevsky in the 19th century, the view of relative mathematics has already been commonsense (Klein, 1980). However, Bloor's originality comes from having extrapolated sociological views of knowledge to mathematics research, and to positing an alternative mathematics from our mathematics at the same level. In recent years, this topic has been the subject of lively discussions in the context of social constructivism (Ernest, 1998). With particular focus on the educational implications of relative construction in mathematics by students, the design of "alternative mathematics" becomes an important key.

"Alternative mathematics," which came about from alternative views, can be conceived by accepting the variability of mathematics. Bloor (1976) illustrates four types of variation in mathematical thought, each of which can be traced back to social causes: divergence of style, meaning, association and standard of cogency. For example, as a first type of variation, Bloor shows the standard early Greek classification of numbers: 1 is not a number because the Greeks saw it as the starting point or generator of numbers. This idea that numbers were units lasted until the 16th century, and we can regard this as alternative mathematics. As a fourth type of variation, he shows the utilization of infinitesimals. In the infinitesimal analysis that flourished under Wallis (J.) or Leibniz (G.W.) in the 17th century, there was a recognition of making infinity applicable to calculations on a par with a number, and not necessarily based on the notion that $1/\infty$ equals 0 by putting the actual sum on the calculus. This can be considered as "alternative mathematics," where the standard of strictness differs from our desire for strict formulas with limitations as in Cauchy (A.L.). The features that such "alternative mathematics" has can mainly be arranged as follows (Bloor, 1974):

a) It would like error or inadequacy, and some of its methods and steps in reasoning would have to violate our sense of logical and cognitive propriety.

b) It might also be embedded in a whole context of purposes and meanings which were utterly alien to our mathematics.

c) The 'errors' in an alternative mathematics would have to be systematic, stubborn and basic. Those features which we deem error would perhaps all be seen to cohere and meaningfully relate to one another by the practitioners of the alternative mathematics.

d) Instead of there being coherence and agreement it could be that lack of consensus was precisely the respect in which the alternative was different to ours. It means cognitive toleration might become a mathematical virtue.

If we look back upon the history of mathematics, there is an abundance of interesting "alternative mathematics." It is well known that Euclid had expressed the view in Book V that the Greeks would only compare homogeneous measurements by forming their quotient. Bochner (1966) points out that the Greek mathematics of Euclid would not have been able to introduce the conceptual product $P \cdot L$ and conceptual ratio P/L for two magnitudes P and L in general when P is one kind of measurement (one unit of measure) and L is another kind of measurement (another unit of measure). Although Greek mathematicians would envisage the proportion $P1 : P2 = L1 : L2$ if L1 and L2 are two values of the same measurement (such as lengths) and P1 and P2 are two values of any other measurement (such as weights), they would not convert the proportions into an equality P1 : L1 = P2 : L2, or P1 · L1 = P2 · L2. While they doubtless had ideas about it, there appeared to be some obstacles in the metaphysical background of their reasoning that kept Greek mathematics from conceptualizing and advancing it in respect of dimensional analysis (Ohara, 2000a, 2000b). Essentially, it is due to the lack of a conception concerning real numbers, but at the same time, Greek mathematics was able to develop a mathematical theory of general physical quantities such as the fifth book of Euclid. This is an illustration of "alternative mathematics." Figure 1 summarizes the relationship between these two kinds of mathematics.

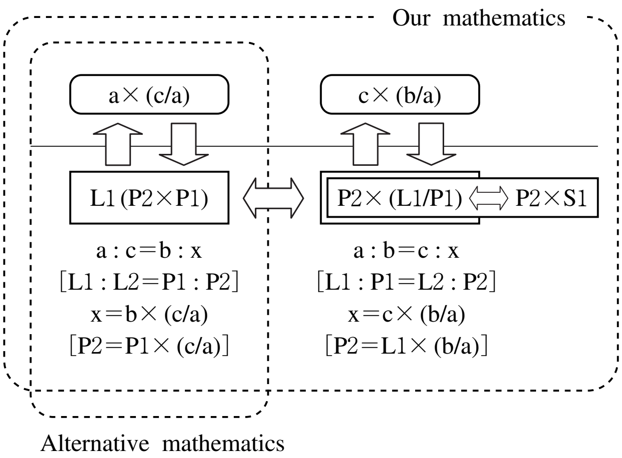

Figure.1

Accordingly, "alternative mathematics" cannot be unitarily interpreted from a modern viewpoint, but it can be identified as a concept that accepts existing values as alternatives on same level of our mathematics.

3. Extension and Significance of "Alternative Mathematics" in a School Context

In this chapter, let us consider the implications of "alternative mathematics" in a school context. Bloor has pointed out the authoritarian character of school education that make students engage in the existing paradigm. However, the discussion on how to consider the existence of "alternative mathematics" follows the problem through a sociological view of knowledge and does not confront school education directly. So, this chapter examines what kind of significance the construction of "alternative mathematics" has in school mathematics. Two viewpoints are submitted as a premise that examines the role of "alternative mathematics."

3. 1. Alternatives for whom

The first viewpoint is concerned with the mutuality of the alternatives and for whom they are aimed. The term "our" that is frequently used in Bloor's arguments means "group," which is sharing a common realization of the most general aspects of the mathematics community. As such, the existence of "alternative mathematics" is specified as a different conformation from "our mathematics" or "school mathematics." However, from the practitioner's view of "alternative mathematics," the direction of "our mathematics" will be heterogeneous mathematics. To apply Bloor's argument in the context of sociological knowledge to school education, it will be helpful to distinguish the student's view and the teacher's view.

Students constitute their naive conceptions both in and out of school, and carry them into the classroom (Resnick, 1987). Occasionally, two mathematical concepts that look the same from the teacher's viewpoint may be viewed completely differently from the student's viewpoint. Likewise, concepts that look the same from the student's viewpoint may be viewed completely differently from the teacher's viewpoint. When the subjectivity gap between the student's mathematics and the teacher's mathematics is resolved, mathematics education can develop wholesomely. This view might be one premise behind education based on social constructivism. Thus, considering the different kind of feelings between students and teacher or among students, "alternative mathematics" is not a settled object that exists outside "our mathematics." Rather, it would be the name from one side in the case of giving heterogeneity mutually to two or more mathematical concepts. This viewpoint of mutuality is important to help make discussing "alternative mathematics" more suitable for mathematics education.

3. 2. Alternatives to what

The second viewpoint is concerned with what the alternatives are to the regulation of mathematical "diversity" (variation). Bloor indicates that the argument for diversity in mathematics is significant only when the system is established socially ; it has firmly entered one culture and could retroact to a social cause (Bloor, 1976). In other words, the existence of “alternative mathematics” can be accepted only when the thought and social cause behind mathematics differ from others. Although we also understand the view holistically, it seems preposterous to treat mathematical diversity only from a global social view in the context of school education. We give width to the scale that constitutes the “society” Bloor refers to, and it is desirable to also accept the value of more local mathematics. That is, it is required to accept the diversity of more localized mathematics as one state of “alternative mathematics” for a more practical treatment of the mathematics the student constructs.

3．3．The Significance of “Alternative Mathematics”

Firstly, we will consider how to overcome the two problems posed in Chapter 1 : “the tendency to regard mathematics as a complete and absolute system” and “lost opportunities for intellectual tolerance in mathematics education.” Concerning the former problem, it almost seems obvious when considering the nature of “alternative mathematics” in Chapter 2. For example, when regarding analysis as “our mathematics,” the nonstandard analysis Robinson proposed in the second half of the 20th century can be considered “alternative mathematics.” This alternative mathematics, called non-standard analysis, introduces ideal elements, hyper-real numbers that include the old real numbers and infinitesimals. Infinitesimals are defined basically as Leibniz attempted (Kline, 1980). Positive or negative infinitesimals are fixed numbers, not variables that approach 0. Robinson constructs this new number system with the same properties as ordinary numbers, but includes infinitesimals. For example, the quotient of infinitesimals dy/dx that exist in the hyper-real system R^*, and dy/dx for y=x2 is 2x+dx, dx is an infinitesimal. Selection of the two systems does not mean a problem of truth but a problem of possibility. “Our mathematics” is just one choice according to social and historical custom. What has been demonstrated in non-standard analysis is that we can recognize “our mathematics” as a human product.

Also, in overcoming the latter problem, the student who thinks infix notation as “our mathematics” faces the reverse Polish notation. For example,

Infix notation (1+2) x (3+4)

Reverse Polish notation 1__2+3__4+x

The reverse Polish notation that adopts the LIFO (Last In First Out) method, has an original merit that can omit specification of an operation order by a parenthesis, and it is adapted to calculators, etc. Let us show the example computation in Reverse Polish notation in figure 2.

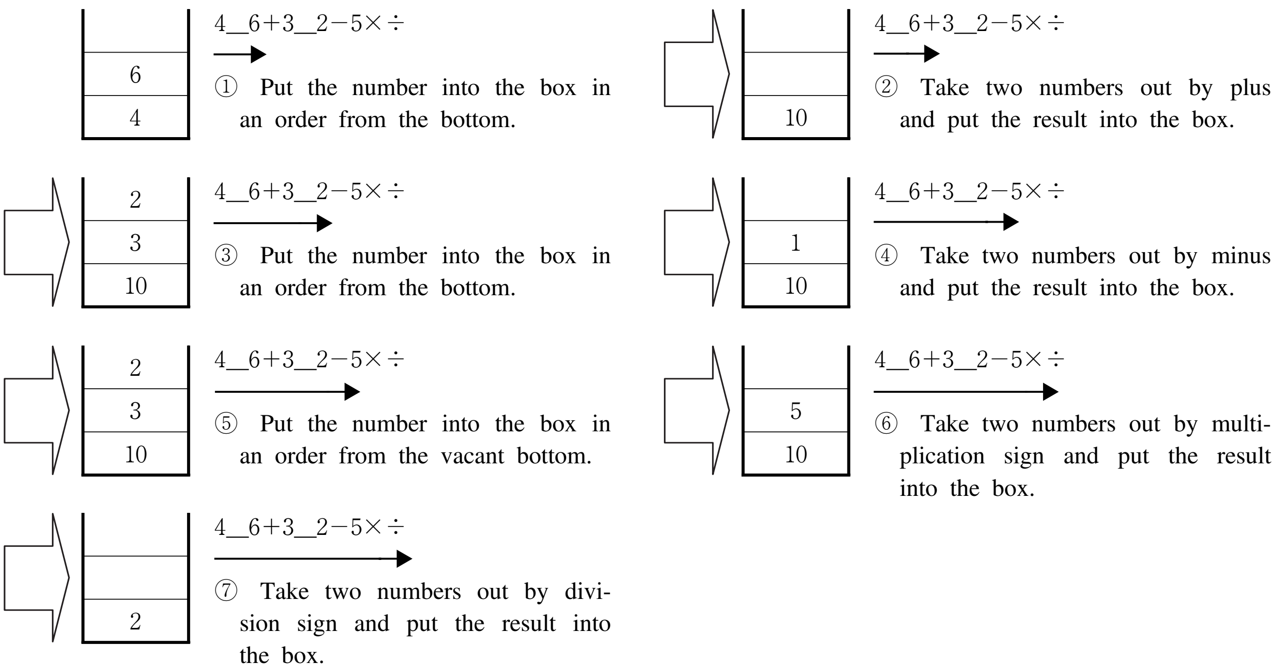

Figure.2

From what has been discussed above, it seems impossible to judge another notation that is different from "our mathematics" as irrational. By handling the reverse Polish notation as "alternative mathematics," students obtain an opportunity to accept each feature mutually without prejudice.

In addition, it points out more positively two significant items in school mathematics that can be considered to constitute "alternative mathematics."

(Ⅰ) Promotion of appreciation

The first point is to know why current mathematics has been supported socioculturally and being able to appreciate this well-defined system. For example, while "alternative mathematics" consists of Roman numerals or Chinese numerals, "our mathematics" consists of Arabic numerals. Shimizu (1995) has pointed out the importance for students of acknowledging the efforts of one's predecessors through comparison of their merits and demerits. If a teacher considers "alternative mathematics" as a medium for reaching a conclusion from some predetermined harmony, there is also a risk that students might consider "alternative mathematics" inferior, such as "the notation system in Arabic numerals is superior to the notation system with Chinese numerals." What has to be highlighted is the necessity for performing "alternative mathematics" in a context that judges the various merits by two or more criteria. In this case, if "alternative mathematics" is compared with "our mathematics," students will have the opportunity to discuss the advantages, such as the ease of writing, the ease of reciting, the ease of calculating and the ease of memorizing from various criteria. Also, students need to know that an agreement (consensus) can be reached in their community (Nakahara, 1994, Ernest, 1998) even based on mathematics with a certain subjective view. By following these processes, they will be able to check the social requirements needed to construct mathematical knowledge.

(Ⅱ) Devolution of intellectual responsibility

The second significance is the provision of an opportunity to devolve intellectual responsibility. Karl Popper has made mention of the idea of intellectual responsibility in relation to the professional ethics that physical philosophers should have. From the standpoint of mathematics philosophy, Ernest (1998) has claimed that mathematics is not neutral, so that mathematics users and creators have a responsibility to society and nature, and school mathematics should reflect this. The devolution of intellectual responsibility is a point that merits special consideration. It is expected that intellectual responsibility will pass from a teacher to students as they take a position of differing views when they engage in mathematics. For example, when students face a non-standard analysis as "alternative mathematics," the students themselves need to construct a new number system based on their consensus. In relation to this problem, perhaps we should consider the teacher's role in the construction of "alternative mathematics." It's possible that a teacher will monopolize the standard of mathematical "correctness." In arriving at a conclusion of lesson, even if various opinions are submitted from students, "alternative mathematics" will seen as secondary and "our mathematics" will be shown as "the canonical one." Under such instruction, you could not expect a self-conscious decision that students opt for to be reached without depending on some external authority. The teacher has to be willing to change the teacher-student relationship from a dependency to a symbiotically relationship between an investigator and a helper.

4. Themes for supporting the construction of "alternative mathematics" in a school context

From what has been said so far, it follows that, at a minimum, "alternative mathematics" has four significances. Moreover, we note three themes that would be posed when supporting a student's construction.

The first problem is the responsibility for intentional education. Schools are organizations that assume the role of reproducing current culture and they intentionally educate with the aim of realizing the vision of that which should exist. Therefore, teachers have a duty to give students optimal content and form. For example, using non-standard analysis to carry out calculations can shake a student's belief in absolute mathematics, but will support the intellectual tolerance of the ideas of others. In addition, it relies on obtaining an opportunity to stimulate an appreciation of standard analysis while simultaneously urging a shift of intellectual responsibility accompanying the construction. However, aside from the construction method, "alternative mathematics" constituted from different viewpoints will be restricted by the curriculum. Furthermore, taking relativism radically and regarding superiority as just a custom or afterthought will deny the normative view of school education. At the very least, we need to be concerned about the rationality of mathematics in a school context.

The second problem is the difficulty students have in accepting diversity. We cannot expect students to easily accept something that is heterogeneous. In addition, the full significance of "alternative mathematics" is something that cannot be accepted immediately. For example, it may be difficult for students trained in standard analysis to accept the handling of infinitesimal numbers in a nonstandard analysis. The same difficulty is seen with the history of mathematics (Cajori, 1919) and the extension of "numbers" such as negative numbers, real numbers and complex numbers. These numbers were not immediately recognized as numbers, and we can see similar historical occurrences, such as non-Euclidean geometry and set theory (Kline, 1980). It takes effort and time to accept the construction or adoption of "alternative mathematics."

The third problem is the possibility that students will be gripped with "Cartesian anxiety" (Bernstein, 1983). When students recognize the world from the viewpoint of relativity, there is an increasing extremely skeptical attitude or a feeling of powerless and a sliding into limitless interpretation by admitting that rationality and truth are undeterministic and equivocal. For example, if students think that anything is allowed regardless of computational systems or regulations, this is not mathematical freedom but intellectual anarchism, characterized as "anything goes" (Feyerabend, 1981). To be sure, at a certain stage it is desirable for students to be exposed to the relative view of mathematics and to understand the axiomatic method. However, at the same time, there needs to be consideration on how to overcome "Cartesian anxiety," which is a feeling of powerlessness.

5. Conclusion

The purpose of this paper is to assemble the significance and problems related to realizing "alternative mathematics" in a school context. As a result, we have pointed to four significant points : 1) overcoming the viewpoint of absolute mathematics, 2) offering the opportunity to raise intellectual tolerance, 3) promotion of appreciation of our mathematics, and 4) devolution of intellectual responsibility. Simultaneously, we have recognized three problems : i) restrictions placed by intentional education, ii) the tremendous amount of mental effort, and iii) the existence of Cartesian anxiety. Based on the significance of "alternative mathematics," if we develop school mathematics with the purpose of supporting proactive learning, it is necessary to provide the opportunity to constitute "alternative mathematics," not to leave it up entirely to the teacher. Lesson planning remains a matter to be discussed. In particular, research on both the history of mathematics and classroom practice would clarify the approach to "alternative mathematics" for students.

References

Barnes, B., Bloor, D., Henry, J. (1996). Scientific knowledge : A sociological analysis. University of Chicago Press.

Bernstein, R.J. (1983). Beyond objectivism and relativism : Science, hermeneutics, and praxis. University of Pennsylvania Press.

Bishop, A. (1985). The social construction of meaning—A significant development for mathematics education? For the Learning of Mathematics 5, 1. pp.24-28.

Bloor, D. (1976). Knowledge and social imagery. The University of Chicago Press, London.

Bloor, D. (1983). Wittgenstein, A social theory of knowledge. Macmillan Press.

Bochner, S. (1966). The role of mathematics in the rise of science. Princeton University Press.

Cajori, F. (1919). A history of mathematics. 2nd ed. New York : Macmillan.

Ernest, P. (1998). Social constructivism as a philosophy of mathematics. State University of New York Press.

Feyerabend, P.K. (1981). Against method : Outline of an anarchistic theory of knowledge.

Gerdes, P. (1996). Ethnomathematics and mathematics education. In A.J. Bishop at al (Eds.), International handbook of mathematics education. Kluwer Academic Publishers. pp.909-943.

Hersh, R. (1997). What is mathematics, really? Oxford University Press.

Kline, M. (1980). Mathematics, the loss of certainty. New York : Oxford University Press.

National Institute for Educational Research (1991). International comparison of mathematics education. Final report of the Second International Mathematics and Science Studies. Dai-ichi Hoki (in Japanese)

National Institute for Educational Research (1997). International comparison of the mathematics education and science education in junior high school. Report of the Third International Mathematics and Science Studies. Toyokan (in Japanese)

Mathisa, A.R. (1992). The Ignorance of Bourbaki. The Mathematical Intelligencer. Vol.14. No.3. Springer-Verlag. pp.4-13.

Minato, S., Hamada, S. (1994) Can Platonic view of Mathemtics Ensure Subjective Learning? -Existence of the Conjunction of View and the Curriculum of Mathematics-. Journal of Japan Society of Mathematical Education, Vol.76, no.3, pp.2-8. (in Japanese)

Nakahara, T. (1994) Development of Constructivism in Mathematics Education ; From Radical to Social Constructivism, Journal of Japan Society of Mathematical Education, Vol.76, no.11, pp.2-11. (in Japanese)

Ohara, Y. (2000a). Epistemological complexity of multiplication and division from the view of dimensional analysis. Wann-Sheng Horng and Fou Lai Lin (eds.), Proceedings of the History and Pedagogy of Mathematics Conference. Vol.1. Taipei. International Study Group on the Relations Between History and Pedagogy of Mathematics. pp. 189-195.

Ohara, Y. (2000b). Note on the multiplicative comparison of magnitude that consists of inhomogeneous magnitudes ; From the view of dimensional analysis. Proceedings of the 24th Annual Meeting, Japan Society for Science Education. pp.251-252.

Ohara, Y. (2002). A Study of Teaching Calculations Involving Four Fundamental Operations, Journal of Japan Society of Mathematical Education, Vol.85, no.10. pp.12-22. (in Japanese)

Resnick, L.B. (1987) Learning in school and out. Educational Researcher. 16.13-20.

Shimizu, S. (1995) Arithmetic that brings out children : Volition to study and appreciation of arithmetic. Shogakukan. (in Japanese)

Vergnaud, G. (1987) About constructivism, A reaction to Hermine Sinclair's and Jeremy Kilpatrick's papers. Proceedings of the 11th Psychology of Mathematics Education. pp.42-48.

On the Significance of Alternative Mathematics in a School context

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The purpose of this study is to discuss the significance and problems of "alternative mathematics" in a school context. For this purpose, we review the thoughts of David Bloor in order to extend the range of the arguments for "alternative mathematics" to education. These discussions lead us to the significance of "Alternative mathematics" : 1) To overcome the view of absolute mathematics, 2) To offer opportunities of raising intellectual tolerance, 3) To promote appreciation of our mathematics, and 4) To devolve intellectual responsibility. At the same time, we are aiming to resolve serious problems : i) Responsibility and purpose of education, ii) Overdoing the level of intellectual effort, and iii) Existence of Cartesian anxiety.