An analysis of advertising campaigns
by distributors and manufacturers

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1．Introduction

Advertisements provide information about the existence and characteristics of products. Some product advertisements increase the demand of unadvertised products. That is, consumers exposed to advertisements sometimes purchase products similar to the advertised product. This purchase behavior results from what are called spillover advertisement effects. Indeed, some manufacturers do not actually advertise their products expecting spillover advertisement effects. In particular, many small electrical manufacturers do not themselves advertise their products. Today, many electrical appliances are sold by distributors. Therefore, consumers can obtain product information from the distributor's homepage. That is, distributors create spillover effects of advertisements. We believe that distributors have a greater advertisement impact today because of consumers' increased Internet search familiarity. Nonetheless, studies on how distributors influence manufacturers' advertising strategies are scarce. In this paper, we highlight the spillover effects of advertising and the impact of distributors' advertisements. This analysis elucidates how distributors influence manufacturers' advertising behavior.

Dorfman and Steiner［1954］，famous for the first analysis of advertising, show that advertising（the resource input）helps shift the demand function to differentiate the product if many manufacturers sell similar products. Nelson［1974］analyzes how product recognition leads to repeat purchases. The author shows that advertising increases sales of all brands in the first step（because of the spillover effect of advertising）while high−quality brands are repeatedly purchased. However, Nelson［1974］did not use a theoretical model to analyze this effect. Kihlstrom and Riordan［1984］filled this gap. Using a theoretical model, they showed that manufacturers who sell high−quality products would benefit if all manufacturers spend the same amount on advertisements that signal quality. Johnsen［1976］shows, using another theoretical model, that advertisements do signal quality. The author reasons that only manufacturers selling high−quality products advertise because only such products sell repeatedly. Dixit and Norman［1978］show that advertisements induce an overall increase in market demand. Roberts and Samuelson［1988］note that "advertising affects primarily the level of market demand rather than the distribution of market shares." By contrast, Kelton and Kelton ［1982］ show that advertising leads primarily to a shift from one brand to another. Bloch and Manceau ［1999］ analyze the effect of persuasive advertising between two competing products by using a model where consumers differ in their tastes. Bloch and Manceau［1999］show that "advertising may induce a decrease in the price of the advertised product, showing that a firm does not necessarily have an incentive to engage in advertising when the two products are sold by different firms." Our analysis shows that distributors influence the advertising strategy of manufacturers.

The rest of the paper is organized as follows. Section 2 describes the spillover effects of advertisements. Section 3 analyses how distributors influence the advertising strategy of manufacturers. Section 4 presents our conclusions and indicates the direction for future research.

2．The basic model

As a starting point, we analyze a simple duopoly model based on the following regimes :
1．Regime 0 : In this regime, two manufacturers determine their output levels so as to maximize their profits without advertisement.
2．Regime 1 : Both manufacturers advertise their products ; no spillover effects occur.
3．Regime N : Only firm 1 advertises its product ; no spillover effects occur.
4．Regime S : Only firm 1 advertises ; spillover effects occur.
5．Regime D : A distributor sells products 1 and 2 but advertises neither.
6．Regime M : The distributor sells both products ; only firm 1 advertises.
7．Regime MD : The distributor sells both products ; firm 1 and the distributor advertise.

2.1 The demand function

We analyze a case in which firm 1 offers product x_1 at price p_1 and firm 2 offers x_2 at p_2. All other products are lumped together as x_0 at a constant quantity and price of 1 .

We suppose that the preferences, that is, the utility functions, of all individuals are identical. Then, we denote the utility function of a representative consumer as U. Assume that an individual's utility depends on the number of units bought of each of the three products, x_0, x_1, and x_2, and that U increases as x_1 or x_2 increases. For simplicity, suppose that a utility function U has the form

$$U(x_0, x_1, x_2) = x_0 + a_1 x_1 + a_2 x_2 - \frac{1}{2}(\beta x_1^2 + \beta x_2^2 + 2\gamma x_1 x_2), \quad (1)$$

where $a_1, a_2, \beta > 0$ and $\beta > \gamma$.

$\beta > \gamma$ implies that the own effect is larger than the cross-effect.

Consumers choose x_1 and x_2 so as to maximize their utility under the budget constraint

$$Y = x_0 + p_1 x_1 + p_2 x_2, \quad (2)$$

where Y is income.
From the first-order conditions, the inverse demand functions are as follows :

$$p_1 = a_1 - \beta x_1 - \gamma x_2, \quad (3)$$
$$p_2 = a_2 - \beta x_2 - \gamma x_1. \quad (4)$$

Now, suppose that advertising informs consumers about the existence, price, and characteristics of x_1 and x_2, and the firms provide consumers the exact information on the products. We suppose that advertising campaigns can increase the demand for the advertised product. That is, advertising campaigns are assumed to induce an upward shift of the demand function. By denoting advertising expenses as K_i, we assume that a_i is a function of K_i, $(da_i / dK_i) > 0$, and $(d^2 a_i / dK_i^2) \le 0$. If the advertising campaigns have no spillover effects, the inverse demand functions of the advertised products are

$$p_1 = a_1(K_1) - \beta x_1 - \gamma x_2, \quad (5)$$
$$p_2 = a_2(K_2) - \beta x_2 - \gamma x_1, \quad (6)$$
where $a_1(K_1) = a_1^0$ if $K_1 = 0$ and $a_2(K_2) = a_2^0$ if $K_2 = 0$.

2. 2　The no-advertising case（Regime 0 ）

We construct the profit function for a no-advertising-campaign case. For simplicity, we suppose that each firm's cost function is linear and not a function of fixed costs, and that the marginal cost is constant. If we define p_i as the unit price minus marginal cost, firm i's profit function is

$$\prod_i = p_i \, x_i. \quad i = 1, \, 2 \quad (7)$$

Substituting the inverse demand functions (5) and (6) into Eq. (7) gives

$$\prod_i = (a_i^0 - \beta x_i - \gamma x_j) \, x_i. \quad i, j = 1, 2, i \neq j \quad (8)$$

Each firm expects its rival to choose the optimal price, and itself chooses x_1 and x_2 so as to maximize its profit. From the first-order conditions, the optimal level of output is as follows :

$$x_1^* = (2\beta a_1^0 - \gamma a_2^0) / (4\beta^2 - \gamma^2), \quad (9)$$
$$x_2^* = (2\beta a_2^0 - \gamma a_1^0) / (4\beta^2 - \gamma^2). \quad (10)$$

The equilibrium prices and the equilibrium profit are

$$p_1^* = \beta \, (2\beta a_1^0 - \gamma a_2^0) / (4\beta^2 - \gamma^2), \quad (11)$$
$$p_2^* = \beta \, (2\beta a_2^0 - \gamma a_1^0) / (4\beta^2 - \gamma^2), \quad (12)$$
$$\prod_1 = \beta \, (2\beta a_1^0 - \gamma a_2^0)^2 / (4\beta^2 - \gamma^2)^2, \quad (13)$$
$$\prod_2 = \beta \, (2\beta a_2^0 - \gamma a_1^0)^2 / (4\beta^2 - \gamma^2)^2. \quad (14)$$

2. 3　Advertising campaigns with no spillover effect（Regime 1 & Regime N）

We consider a case in which the advertising campaign has no spillover effect. If each firm is assumed to advertise its product, consumers recognize the advertised product, and its demand increases. In this game, the strategy of each firm is implemented in two steps. Each firm is assumed to advertise its product in the first step, and determine the output level that maximizes its profit in the second step. Then, the maximization problems that each firm faces in the second step are

$$\prod_1 = (a_1 \, (K_1 \,) - \beta x_1 - \gamma x_2) \, x_1 \,, \quad (15)$$
$$\prod_2 = (a_2 \, (K_2 \,) - \beta x_2 - \gamma x_1) \, x_2 \,, \quad (16)$$

The sub-game equilibrium output is

$$x_1^{**} = (2 \, \beta a_1(K_1) - \gamma a_2^0) / (4\beta^2 - \gamma^2), \quad (17)$$
$$x_2^{**} = (2 \, \beta a_2(K_2) - \gamma a_1^0) / (4\beta^2 - \gamma^2), \quad (18)$$

The equilibrium prices are

$$p_1^{**} = \beta \, (2 \, \beta a_1(K_1) - \gamma a_2^0) / (4\beta^2 - \gamma^2), \quad (19)$$
$$p_2^{**} = \beta \, (2 \, \beta a_2(K_2) - \gamma a_1^0) / (4\beta^2 - \gamma^2), \quad (20)$$

Next, we suppose that it is only firm 1 that advertises its product. All other assumptions remain the same. In the first step, firm 1 is assumed to regard the equilibrium output in the second step as given and determine its advertising expenses so as to maximize its profit. In the second step, each firm regards the advertising expenses of firm 1 as given (it is assumed to know firm 1's advertising expenses and the

effect of advertisement) and determines its output so as to maximize its profit. Let K_i^2 be firm i's advertising expenses. Then, the maximization problem that firm 1 faces is

$$\prod_1 = p_1^N x_1^N - K_1^2, \quad (21)$$

where $\alpha_i(K_i) = \alpha_i^0 + \varkappa K_i$, $\varkappa$ is a constant, and $0 < \varkappa < 1$. The first–order condition gives

$$K_1^N = \varkappa (4\beta^2 - \gamma^2) x_1^* / 2 (4\beta^2 - \gamma^2 - \varkappa^2 \beta) > 0. \quad (22)$$

Eqs. (15) and (16) give the equilibrium output, price, and profit :

$$x_1^N = x_1^* + 2\varkappa\beta K_1^N / (4\beta^2 - \gamma^2), \quad (23)$$
$$x_2^N = x_2^* - \varkappa\gamma K_1^N / (4\beta^2 - \gamma^2), \quad (24)$$
$$p_1^N = p_1^* + 2\varkappa\beta^2 K_1^N / (4\beta^2 - \gamma^2), \quad (25)$$
$$p_2^N = p_2^* - \varkappa\beta\gamma K_1^N / (4\beta^2 - \gamma^2), \quad (26)$$
$$\prod_1^N = \prod_1^* + 2\varkappa\beta^2 x_1^* K_1^N / (4\beta^2 - \gamma^2), \quad (27)$$
$$\prod_2^N = p_2^N x_2^N. \quad (28)$$

Proposition 1 (1) If $x_1^N > x_1^*$, then $\gamma \geq 0 \Rightarrow x_2^N \leq x_2^*$, $\gamma < 0 \Rightarrow x_2^N > x_2^*$
             (2) If $p_1^N > p_1^*$, then $\gamma \geq 0 \Rightarrow p_2^N \leq p_2^*$, $\gamma < 0 \Rightarrow p_2^N > p_2^*$
             (3) If $\prod_1^N > \prod_1^*$, then $\gamma \geq 0 \Rightarrow \prod_2^N \leq \prod_2^*$, $\gamma < 0 \Rightarrow \prod_2^N > \prod_{2*}$

Proposition 1 indicates that the advertising campaign of firm 1 increases the demand of the product, and in turn the output, price, and profit, of firm 1. Then, the equilibrium output, price, and profit of the substitutes decrease, whereas the equilibrium output, price, and profit of complements increase.

2.4 Advertising campaigns with spillover effects (Regime S)

Next, we consider a case in which the advertising campaigns have spillover effects. All other assumptions remain the same. If the advertising campaigns have spillover effects, firm 1's campaign would increase the demand for firm 2's product as well in addition to its own.

$$p_1 = \alpha_1(K_1, K_2) - \beta x_1 - \gamma x_2 \quad (29)$$
$$p_2 = \alpha_1(K_1, K_2) - \beta x_2 - \gamma x_1 \quad (30)$$

Now, we define the spillover rate of the advertising campaigns as the percentage by which the advertising campaign for product 1 increases the demand for product 2. We denote the rate as ζ, assuming $0 < \zeta < 1 - \zeta$. This assumption implies that the spillover effect of the advertising campaign is less than the increase in the demand for the advertised product. The spillover rate of advertising for product 2 is estimated similarly. Furthermore, we assume that $\alpha_1(K_1, K_2) = \alpha_1^0 + \varkappa[(1-\zeta)K_1 + \zeta K_2]$ and $\alpha_2(K_1, K_2) = \alpha_2^0 + \varkappa[(1-\zeta)K_2 + \zeta K_1]$. Again, we consider a case in which only firm 1 advertises product 1. When only firm 1 advertises its product in the first step, the profit that each firm faces in the second step is

$$\prod_1 = (\alpha_1^0 + \varkappa(1-\zeta)K_1 - \beta x_1 - \gamma x_2) x_1, \quad (31)$$
$$\prod_2 = (\alpha_2^0 + \varkappa\zeta K_1 - \beta x_2 - \gamma x_1) x_2. \quad (32)$$

From the first–order conditions, the sub–game equilibrium output in the second step is as follows :

$$x_1^S = \{2\beta[\alpha_1^0 + \varkappa(1-\zeta)K_1] - \gamma(\alpha_2^0 + \varkappa\zeta K_1)\} / (4\beta^2 - \gamma^2), \quad (33)$$

$$x_2^S = \{2\beta\,(a_2^0 + \varkappa\zeta K_1\,) - \gamma\,[a_1^0 + \varkappa\,(1-\zeta)\,K_1\,]\}\,/\,(4\beta^2 - \gamma^2\,). \quad \text{(34)}$$

The equilibrium prices are

$$p_1^S = \beta\,\{2\beta\,[a_1^0 + \varkappa\,(1-\zeta)\,K_1\,] - \gamma\,(a_2^0 + \varkappa\zeta K_1\,)\}\,/\,(4\beta^2 - \gamma^2\,), \quad \text{(35)}$$
$$p_2^S = \beta\,\{2\beta\,(a_2^0 + \varkappa\zeta K_1\,) - \gamma\,[a_1^0 + \varkappa\,(1-\zeta)\,K_1\,]\}\,/\,(4\beta^2 - \gamma^2\,). \quad \text{(36)}$$

Then, the problem firm 1 faces is

$$\Pi_1 = p_1^S\, x_1^S - K_1^2. \quad \text{(37)}$$

From the first-order condition,

$$K_1^S = \varkappa\,(1-\zeta)\,(4\beta^2 - \gamma^2\,)\,x_1^*\,/\,\{2\,(4\beta^2 - \gamma^2) - \varkappa^2\,(1-\zeta)\,[2\beta\,(1-\zeta) - \gamma\zeta]\} > 0. \quad \text{(38)}$$

The equilibrium output, price, and profit are

$$x_1^S = x_1^* + [2\beta\,(1-\zeta) - \gamma\zeta\,]\,\varkappa K_1^S\,/\,(4\beta^2 - \gamma^2\,), \quad \text{(39)}$$
$$x_2^S = x_2^* - [\gamma\,(1-\zeta) - 2\beta\zeta\,]\,\varkappa K_1^S\,/\,(4\beta^2 - \gamma^2\,), \quad \text{(40)}$$
$$p_1^S = p_1^* + \beta\,[2\beta\,(1-\zeta) - \gamma\zeta\,]\,\varkappa K_1^S\,/\,(4\beta^2 - \gamma^2\,), \quad \text{(41)}$$
$$p_2^S = p_2^* - \beta\,[\gamma\,(1-\zeta) - 2\beta\zeta\,]\,\varkappa K_1^S\,/\,(4\beta^2 - \gamma^2\,), \quad \text{(42)}$$
$$\Pi_1^S = \Pi_1^* + \beta\,[2\beta\,(1-\zeta) - \gamma\zeta\,]\,x_1^*\varkappa K_1^S\,/\,(4\beta^2 - \gamma^2\,), \quad \text{(43)}$$
$$\Pi_2^S = p_2^S\, x_2^S. \quad \text{(44)}$$

Proposition 2 Equilibrium output, price, and profit comparisons with spillover effects of advertising :

$$x_1^S > x_1^*,\ p_1^S > p_1^*,\ \Pi_1^S > \Pi_1^*$$
$$\gamma \le 0 \Rightarrow x_2^S > x_2^*, p_2^S > p_2^*, \Pi_2^S > \Pi_2^*$$
$$\gamma > 0, \zeta \ge \gamma/\,(2\beta + \gamma) \Rightarrow x_2^S \ge x_2^*, p_2^S \ge p_2^*, \Pi_2^S \ge \Pi_2^*$$
$$\gamma > 0, \zeta < \gamma/\,(2\beta + \gamma) \Rightarrow x_2^S < x_2^*, p_2^S < p_2^*, \Pi_2^S < \Pi_2^*$$

Proposition 2 indicates that an advertising campaign increases the demand for the advertised product even if it has spillover effects. Furthermore, the proposition indicates that an advertising campaign with a high rate of spillover increases the demand for complements and substitutes of the advertised product. Next, we analyze how the equilibrium changes if advertising has spillover effects.

Proposition 3 The change in equilibrium with spillover effects of advertising :

$$x_1^N > x_1^S,\ p_1^N > p_1^S,\ \Pi_1^N > \Pi_1^S$$
$$\gamma > 0 \Rightarrow x_2^S > x_2^N,\ p_2^S > p_2^N,\ \Pi_2^S > \Pi_2^N$$
$$\gamma < 0,\ K_1^N/K_1^S \le [\gamma\,(1-\zeta) - 2\beta_1\,\zeta\,]/\gamma \Rightarrow x_2^S \ge x_2^N,\ p_2^S \ge p_2^N,\ \Pi_2^S \ge \Pi_2^N$$
$$\gamma < 0,\ K_1^N/K_1^S > [\gamma\,(1-\zeta) - 2\beta_1\,\zeta\,]/\gamma \Rightarrow x_2^S < x_2^N,\ p_2^S < p_2^N,\ \Pi_2^S < \Pi_2^N$$
$$K_1^S < K_1^N$$

Proof.
$$K_1^N - K_1^S = \frac{\varkappa\beta\zeta\mu\,\{2\beta\mu^2 + \mu^2 + 2\varkappa^2\beta^2\gamma + 4\varkappa^2\beta^2\,[2\beta\,(1-\zeta) - \gamma\zeta\,]\}}{(\mu^2 - 4\varkappa^2\beta^2\,)\,\{\mu^2 - \varkappa^2\beta\,[2\beta\,(1-\zeta) - \gamma\zeta\,]^2\,\}} > 0,$$
such that $\mu = 4\beta^2 - \gamma^2$.

$$x_1^S - x_1^N = \varkappa \left[2\beta \left(K_1^S - K_1^N \right) - \zeta K_1^S \left(2\beta + \gamma \right) \right] / \mu < 0$$
$$\qquad\qquad\quad (-) \qquad\qquad (+)$$
$$p_1^S - p_1^N = \beta \left(x_1^S - x_1^N \right) < 0$$
$$x_2^S - x_2^N = \alpha \left[\gamma \left(K_1^N - K_1^S \right) + \zeta K_1^S \left(2\beta + \gamma \right) \right] / \mu, \ p_2^S - x_2^N = \beta \left(x_2^S - x_2^N \right)$$
From $\prod_2^S - \prod_2^N = \beta \left(x_2^{S2} - x_2^{N2} \right),$
$$\gamma > 0 \Rightarrow x_2^S > x_2^N, \ p_2^S > p_2^N, \ \prod_2^S > \prod_2^N,$$
$$\gamma < 0, \ K_1^N / K_1^S \leq \left[\gamma \left(1 - \zeta \right) - 2\beta_1 \zeta \right] / \gamma \Rightarrow x_2^S \geq x_2^N, \ p_2^S \geq p_2^N, \ \prod_2^S \geq \prod_2^N,$$
$$\gamma < 0, \ K_1^N / K_1^S > \left[\gamma \left(1 - \zeta \right) - 2\beta_1 \zeta \right] / \gamma \Rightarrow x_2^S < x_2^N, \ p_2^S < p_2^N, \ \prod_2^S < \prod_2^N.$$

Proposition 3 indicates that the increase in the equilibrium output, price, and profit of the advertised product is greater without spillover effects than with such effects. The proposition indicates that the equilibrium output, price, and profit of the substitutes of the advertised product increase if advertising has spillover effects. With spillover effects, the advertising campaign increases the demand for the rival firm's product, with the decrease in the demand−increasing effects of own−product advertisements.

2. 5 The spillover effects of advertising and social welfare
Next, we consider how the spillover effects of advertising of product 1 alter social welfare. First, let us define the consumer surplus of the unadvertised product of firms 1 and 2 as CS_1^* and CS_2^*, respectively.

$$CS_1^* = (1/2)(\alpha_1^0 - p_1^*) x_1^* = (1/2)(\alpha_1^0 - \beta x_1^*) x_1^* \quad (45)$$
$$CS_2^* = (1/2)(\alpha_2^0 - p_2^*) x_2^* = (1/2)(\alpha_2^0 - \beta x_2^*) x_2^* \quad (46)$$

Second, when only product 1 is advertised, with no spillover effects, products 1 and 2 respectively yield consumer surpluses (CS_1^N, CS_2^N) as follows based on Eqs. (22) to (26):

$$CS_1^N = (1/2)(\alpha_1^0 \varkappa K_1^N - p_1^N) x_1^N$$
$$= (1/2)(\alpha_1^0 + \varkappa K_1^N - \beta x_1^N) x_1^N$$
$$CS_1^* + \frac{\varkappa K_1^N \left[2\beta (\alpha_1^0 - \beta x_1^*)(4\beta^2 - \gamma^2) + (2\beta^2 - \gamma^2)\{(4\beta^2 - \gamma^2) x_1^* + 2\varkappa\beta K_1^N\} \right]}{2(4\beta^2 - \gamma^2)^2}, \quad (47)$$
$$CS_2^N = CS_2^* - \frac{\varkappa K_1^N \left[\gamma (\alpha_2^0 - \beta x_2^*)(4\beta^2 - \gamma^2) + \beta\gamma\{(4\beta^2 - \gamma^2) x_2^* - \varkappa\beta K_1^N\} \right]}{2(4\beta^2 - \gamma^2)^2}. \quad (48)$$

Third, with the spillover effects of firm 1's advertising campaigns, products 1 and 2 respectively yield consumer surpluses (CS_1^S, CS_2^S) as follows based on Eqs. (38) to (42):

$$CS_1^S = (1/2)(\alpha_1^0 - p_1^S) x_1^S$$
$$= (1/2)(\alpha_1^0 - \beta x_1^S) x_1^S$$
$$= CS_1^* + \frac{\varkappa K_1^S \left[(2\beta^2 - \gamma^2)(1 - \zeta) - \gamma\zeta \right] (4\beta^2 - \gamma^2)}{2(4\beta^2 - \gamma^2)^2} x_1^*$$
$$+ \frac{\varkappa K_1^S \{ (\alpha_1^0 - \beta x_1^*) [2\beta (1 - \zeta) - \gamma\zeta](4\beta^2 - \gamma^2) + [(2\beta^2 - \gamma^2)(1 - \zeta) - \gamma\zeta][2\beta (1 - \zeta) - \gamma\zeta]\varkappa K_1^S \}}{2(4\beta^2 - \gamma^2)^2}, \quad (49)$$
$$CS_2^S = (1/2)(\alpha_2^0 - p_2^S) x_2^S$$
$$= (1/2)(\alpha_2^0 - \beta x_2^S) x_2^S$$
$$= CS_2^* + \frac{\varkappa K_1^S \zeta (4\beta^2 - \gamma^2)}{2(4\beta^2 - \gamma^2)^2} x_2^*$$
$$+ \frac{\varkappa K_1^S \left[\gamma (1 - \zeta) - 2\beta\zeta \right] \{ [\varkappa K_1^S [\beta\gamma (1 - \zeta) - (6\beta^2 - \gamma^2)\zeta] - \alpha_2^0 (4\beta^2 - \gamma^2) \}}{2(4\beta^2 - \gamma^2)^2}. \quad (50)$$

Proposition 4. Comparison of consumer surplus :

$$CS_1^N > CS_1^S > CS_1^*$$
$$\gamma > 0, \zeta \geq \gamma(2\beta+\gamma) \Rightarrow CS_2^S \geq CS_2^* \geq CS_2^N$$
$$\gamma > 0, \zeta < \gamma(2\beta+\gamma) \Rightarrow CS_2^S < CS_2^* < CS_2^N$$
$$\gamma = 0 \Rightarrow CS_2^S \geq CS_2^* = CS_2^N$$
$$\gamma < 0, \phi \geq 0 \Rightarrow CS_2^N > CS_2^S > CS_2^*$$
$$\text{such that } \phi = K_1^N / K_1^S - [\gamma(1-\zeta) - 2\beta\zeta]$$

Proposition 4 is proved in the appendix. Proposition 4 indicates that the advertising campaign for product 1 increases the consumer surplus of product 1. The less the spillover effects of advertising, the greater is the consumer surplus of the advertised product. The consumer surplus of the substitutes of the advertised product increases with no advertising, since the advertising campaign decreases the demand for substitutes of the advertised product. However, the advertising campaign can increase the consumer surplus of the substitutes of the advertised product with a high spillover rate of advertising.

Proposition 5 Comparison of producer surplus :

$$\gamma \geq 0, \zeta \geq \gamma(2\beta+\gamma) \Rightarrow \prod_2^S \geq \prod_2^* \geq \prod_2^N$$
$$\gamma \geq 0, \zeta < \gamma(2\beta+\gamma) \Rightarrow \prod_2^* \geq \prod_2^S \geq \prod_2^N$$
$$\gamma < 0, \phi \geq 0 \Rightarrow \prod_2^N \geq \prod_2^S \geq \prod_2^*$$
$$\text{such that } \phi = K_1^N / K_1^S - [\gamma(1-\zeta) - 2\beta\zeta]$$

Proposition 5 indicates that the advertising campaign decreases the profits from substitutes of the advertised product. However, the advertising campaign can increase the rival's profit with a high spillover rate of advertising. Thus, the advertising campaign can increase not only consumer surplus and producer surplus of the advertised product, but also consumer surplus and producer surplus of the substitutes of the advertised product with a high spillover rate of advertising.

3. Advertising campaign by the distributor

3.1 Market with two manufactures and a distributor (Regime D)

Next, we consider a market in which a distributor sells both products 1 and 2. We suppose that the industry structure is described by a Stackelberg–type leader–follower model, with firms 1 and 2 playing leader roles and the distributor acting as a follower. That is, firms 1 and 2 determine the wholesale prices so as to maximize their profits in the first step, and the distributor determines the retail prices so as to maximize the profit in the second step. Then, the demand functions are

$$p_1 = a_1 - \beta x_1 - \gamma x_2 , \quad (3)$$
$$p_2 = a_2 - \beta x_2 - \gamma x_1 . \quad (4)$$

Let r_1 and r_2 be the wholesale prices of products 1 and 2, respectively. Then the distributor's profit in the second step is

$$\Omega = (p_1 - r_1) x_1 + (p_2 - r_2) x_2 ,$$
$$= (a_1 - \beta x_1 - \gamma x_2 - r_1) x_1 + (a_2 - \beta x_2 - \gamma x_1 - r_2) x_2 . \quad (51)$$

From the first–order condition, the sub–game equilibrium output in the second step is as follows :

$$x_1^D = [\beta\,(\alpha_1 - r_1) - \gamma\,(\alpha_2 - r_2)]/2\,(\beta^2 - \gamma^2),\quad (52)$$
$$x_2^D = [\beta\,(\alpha_2 - r_2) - \gamma\,(\alpha_1 - r_1)]/2\,(\beta^2 - \gamma^2).\quad (53)$$

Each firm's profit in the first step is

$$\prod_1 = r_1\,x_1^D = r_1\,[\beta\,(\alpha_1 - r_1) - \gamma\,(\alpha_2 - r_2)]/2\,(\beta^2 - \gamma^2),\quad (54)$$
$$\prod_2 = r_2\,x_2^D = r_2\,[\beta\,(\alpha_2 - r_2) - \gamma\,(\alpha_1 - r_1)]/2\,(\beta^2 - \gamma^2).\quad (55)$$

From the first-order conditions, the sub-game equilibrium wholesale prices in the first step is

$$r_1^D = [(2\beta^2 - \gamma^2)\,\alpha_1 - \beta\gamma\alpha_2\,]/(4\beta^2 - \gamma^2),\quad (56)$$
$$r_2^D = [(2\beta^2 - \gamma^2)\,\alpha_2 - \beta\gamma\alpha_1\,]/(4\beta^2 - \gamma^2).\quad (57)$$

From Eqs. (34) and (35), we obtain the equilibrium output and retail prices as follows :

$$x_1^D = \beta r_1^D/2\,(\beta^2 - \gamma^2) = \beta\,[(2\beta^2 - \gamma^2)\,\alpha_1 - \beta\gamma\alpha_2\,]/(2\beta^2 - \gamma^2)(4\beta^2 - \gamma^2),\quad (58)$$
$$x_2^D = \beta r_2^D/2\,(\beta^2 - \gamma^2) = \beta\,[(2\beta^2 - \gamma^2)\,\alpha_2 - \beta\gamma\alpha_1\,]/(2\beta^2 - \gamma^2)(4\beta^2 - \gamma^2),\quad (59)$$
$$p_1^D = [2\,(3\beta^2 - \gamma^2)\,\alpha_1 - \beta\gamma\alpha_2\,]/2\,(4\beta^2 - \gamma^2),\quad (60)$$
$$p_2^D = [2\,(3\beta^2 - \gamma^2)\,\alpha_2 - \beta\gamma\alpha_1\,]/2\,(4\beta^2 - \gamma^2).\quad (61)$$

3. 2 Where firm 1 advertises but the distributor does not (Regime M)

Next, we suppose that firm 1 advertises in the first step, each firm determines the wholesale prices so as to maximize its profits in the second step, and the distributor determines the retail prices so as to maximize the profit in the third step. All other assumptions remain the same. Then, advertising by firm 1 is assumed to increase only the demand for product 1. Let K_1 be the advertising expense and $\alpha_1 = \alpha_1(K_1),\ \partial\alpha_1/\partial K_1 > 0$. Then, Eq. (3) is rewritten as

$$p_1 = \alpha_1(K_1) - \beta x_1 - \gamma x_2,\quad (62)$$
$$\text{such that } \alpha_1(K_1) = \alpha_1^0 \text{ at } K_1 = 0.$$

Then, the distributor's profit in the third step is

$$\Omega = (p_1 - r_1)\,x_1 + (p_2 - r_2)\,x_2$$
$$= (\alpha_1(K_1) - \beta x_1 - \gamma x_2 - r_1)\,x_1 + (\alpha_2 - \beta x_2 - \gamma x_1 - r_2)\,x_2.\quad (63)$$

From the first-order conditions, the sub-game equilibrium output in the third step is as follows :

$$x_1^M = [\beta\,(\alpha_1(K_1) - r_1) - \gamma\,(\alpha_2 - r_2)]/2\,(\beta^2 - \gamma^2),\quad (64)$$
$$x_2^M = [\beta\,(\alpha_2 - r_2) - \gamma\,(\alpha_1(K_1) - r_1)]/2\,(\beta^2 - \gamma^2).\quad (65)$$

Then, each firm's profit in the second step is

$$\prod_1 = r_1\,x_1^M = r_1\,[\beta\,(\alpha_1(K_1) - r_1) - \gamma\,(\alpha_2 - r_2)]/2\,(\beta^2 - \gamma^2),\quad (66)$$
$$\prod_2 = r_2\,x_2^M = r_2\,[\beta\,(\alpha_2 - r_2) - \gamma\,(\alpha_1(K_1) - r_1)]/2\,(\beta^2 - \gamma^2).\quad (67)$$

From the first-order conditions, the sub-game equilibrium wholesale prices in the second step are as follows :

$$r_1^M = [(2\beta^2 - \gamma^2)\alpha_1(K_1) - \beta\gamma\alpha_2]/(4\beta^2 - \gamma^2), \quad (68)$$
$$r_2^M = [(2\beta^2 - \gamma^2)\alpha_2 - \beta\gamma\alpha_1(K_1)]/(4\beta^2 - \gamma^2). \quad (69)$$

From Eqs. (46) and (47), the equilibrium output and prices are as follows:

$$x_1^M = \beta\gamma_1^M/2(\beta^2 - \gamma^2) = \beta[(2\beta^2 - \gamma^2)\alpha_1(K_1) - \beta\gamma\alpha_2]/2(\beta^2 - \gamma^2)(4\beta^2 - \gamma^2), \quad (70)$$
$$x_2^M = \beta\gamma_2^M/2(\beta^2 - \gamma^2) = \beta[(2\beta^2 - \gamma^2)\alpha_2 - \beta\gamma\alpha_1(K_1)]/2(\beta^2 - \gamma^2)(4\beta^2 - \gamma^2), \quad (71)$$
$$p_1^M = [2(3\beta^2 - \gamma^2)\alpha_1(K_1) - \beta\gamma\alpha_2]/2(4\beta^2 - \gamma^2), \quad (72)$$
$$p_2^M = [2(3\beta^2 - \gamma^2)\alpha_2 - \beta\gamma\alpha_1(K_1)]/2(4\beta^2 - \gamma^2), \quad (73)$$

If K_1^2 represents the advertising expenses of firm 1, the profit of firm 1 is

$$\Pi_1 = r_1^M x_1^M - K_1^2$$
$$= \beta r_1^{M2}/2(\beta^2 - \gamma^2) - K_1^2$$
$$= \beta[(2\beta^2 - \gamma^2)\alpha_1(K_1) - \beta\gamma\alpha_2]^2/2(\beta^2 - \gamma^2)(4\beta^2 - \gamma^2)^2 - K_1^2. \quad (74)$$

From the first-order condition, if $\alpha_1(K_1) = \alpha_1^0 + \varkappa K_1$, the equilibrium advertising expense is

$$K_1^M = \frac{\beta\varkappa(2\beta^2 - \gamma^2)[(2\beta^2 - \gamma^2)\alpha_1^0 - \beta\gamma\alpha_2^0]}{2(\beta^2 - \gamma^2)(4\beta^2 - \gamma^2)^2 - \beta\varkappa^2(2\beta^2 - \gamma^2)^2}. \quad (75)$$

From Eq. (75), the equilibrium profit of firm 1 is

$$\Pi_1^M = \Pi_1^D + \frac{(2\beta^2 - \gamma^2)\varkappa K_1^M x_1^D}{4\beta^2 - \gamma^2}. \quad (76)$$

Proposition 6 Comparison between equilibrium quantities under Regimes D (distributor sells products 1 and 2; none advertises) and M (distributor sells products 1 and 2; firm 1 advertises):

$$x_1^D < x_1^M, \; r_1^D < r_1^M, \; p_1^D < p_1^M, \; \Pi_1^D < \Pi_1^M$$
$$\gamma > 0 \Rightarrow x_2^D > x_2^M, \; r_2^D > r_2^M, \; p_2^D > p_2^M, \; \Pi_2^D > \Pi_2^M$$
$$\gamma < 0 \Rightarrow x_2^D < x_2^M, \; r_2^D < r_2^M, \; p_2^D < p_2^M, \; \Pi_2^D < \Pi_2^M$$

Proposition 6 indicates that when a distributor sells products 1 and 2, the advertising campaign for product 1 increases its demand and, in turn, the equilibrium output, price, and profit of firm 1. Further, the campaign decreases the demand and, in turn, the equilibrium output, price, and profit of the substitutes for the advertised product.

3.3 Where both firm 1 and distributor advertise (Regime MD)

We consider a case in which both firm 1 and the distributor advertise in the second step. All other assumptions remain the same. Suppose firm 1 advertises in the first step, both firms determine the wholesale price so as to maximize their profits in the second step and the distributor advertises both products 1 and 2 in the third step and determines the retail prices so as to maximize profits in the fourth step. Furthermore, firm 1's advertisement is assumed to increase the demand for product 1, while the distributor's advertisement is assumed to increase the demand for both products 1 and 2. That is, letting K_1 be firm 1's and K_D the distributor's advertisement, $\alpha_1 = \alpha_1(K_1, K_D)$, $\alpha_2 = \alpha_2(K_D)$, $\partial\alpha_1/\partial K_1 > 0$, $\partial\alpha_1/\partial K_D > 0$, and $\partial\alpha_2/\partial K_D > 0$. However, if $K_1 = 0$, $K_D = 0$, then $\alpha_1(K_1, K_D) = \alpha_1^0$ and $\alpha_2(K_D) = \alpha_2^0$. Then, Eq. (3) is rewritten as

$$p_1 = a_1(K_1, K_D) - \beta x_1 - \gamma x_2, \quad (77)$$
$$p_2 = a_2(K_D) - \beta x_2 - \gamma x_1. \quad (78)$$

Then, the profit of the distributor in the fourth step is

$$\Omega = (p_1 - r_1) x_1 + (p_2 - r_2) x_2$$
$$= (a_1(K_1, K_D) - \beta x_1 - \gamma x_2 - r_1) x_1 + (a_2(K_D) - \beta x_2 - \gamma x_1 - r_2) x_2. \quad (79)$$

From the first–order conditions, the sub–game equilibrium sales amount is

$$x_1^{MD} = [\beta (a_1(K_1, K_D) - r_1) - \gamma (a_2(K_D) - r_2)] / 2(\beta^2 - \gamma^2), \quad (80)$$
$$x_2^{MD} = [\beta (a_2(K_D) - r_2) - \gamma (a_1(K_1, K_D) - r_1)] / 2(\beta^2 - \gamma^2). \quad (81)$$

If K_D^2 represents the advertising expenses, the maximization problem of the distributor in the third step is

$$\Omega = (p_1 - r_1) x_1^{MD} + (p_2 - r_2) x_2^{MD} - K_D^2. \quad (82)$$

If $a_1 (K_1, K_D) a_1^0 + \varkappa K_1 + m K_D$ ($0 < \varkappa < 1$, $0 < m < 1$, $\varkappa$, and m are constant), $a_2 (K_D) = a_2^0 + m K_D$. From the first–order condition, we obtain the sub–game equilibrium advertising expenses of the distributor :

$$K_D^{MD} = m (x_1^{MD} + x_2^{MD}) / 2. \quad (83)$$

From Eq. (83), we rewrite (80) and (81) as

$$x_1^{MD} = [B (a_1(K_1) - r_1) - H (a_2^0 - r_2)] / 2 (B^2 - H^2), \quad (84)$$
$$x_2^{MD} = [B (a_2^0 - r_2) - H (a_1(K_1) - r_1)] / 2 (B^2 - H^2) \quad (85)$$
$$\text{such that } B = 4\beta - m^2, H = 4\gamma - m^2.$$

The profit of each firm in the second step is

$$\prod_1 = r_1 x_1^{MD} = r_1 [B (a_1(K_1) - r_1) - H (a_2^0 - r_2)] / 2 (B^2 - H^2), \quad (86)$$
$$\prod_2 = r_2 x_2^{MD} = r_2 [B (a_2^0 - r_2) - H (a_1(K_1) - r_1)] / 2 (B^2 - H^2). \quad (87)$$

From the first–order conditions, the sub–game equilibrium wholesale prices are

$$r_1^{MD} = [(2B^2 - H^2) a_1(K_1) - BH a_2^0] / (4B^2 - H^2), \quad (88)$$
$$r_2^{MD} = [(2B^2 - H^2) a_2^0 - BH a_1(K_1)] / (4B^2 - H^2). \quad (89)$$

From Eqs. (65) and (66), we obtain the equilibrium output and prices :

$$x_1^{MD} = B [(2B^2 - H^2) a_1(K_1) - BH a_2^0] / 2 (B^2 - H^2)(4B^2 - H^2), \quad (90)$$
$$x_2^{MD} = B [(2B^2 - H^2) a_2^0 - BH a_1(K_1)] / 2 (B^2 - H^2)(4B^2 - H^2), \quad (91)$$
$$p_1^{MD} = B [2(3B^2 - H^2) a_1(K_1) - BH a_2^0] / 2 (4B^2 - H^2), \quad (92)$$
$$p_2^{MD} = B [2(3B^2 - H^2) a_2^0 - BH a_1(K_1)] / 2 (4B^2 - H^2). \quad (93)$$

If the advertising expense of firm 1 is given as K_1^2, its profit is

$$\prod_1 = r_1^{MD} x_1^{MD} - K_1^2$$

$$= \mathrm{B} r_1^{MD2}/2\,(\mathrm{B}^2 - \mathrm{H}^2) - K_1^2$$

$$= \mathrm{B}\,[\,(2\mathrm{B}^2 - \mathrm{H}^2)\,\alpha_1(K_1) - \mathrm{BH}\alpha_2^0\,]^2 \,/\, 2\,(\mathrm{B}^2 - \mathrm{H}^2)\,(4\mathrm{B}^2 - \mathrm{H}^2) - K_1^2. \quad (94)$$

From the first-order condition, the equilibrium advertising expense is

$$K_1^{MD} = \frac{\mathrm{B}\varkappa\,(2\mathrm{B}^2 - \mathrm{H}^2)\,[\,(2\mathrm{B}^2 - \mathrm{H}^2)\,\alpha_1^0 - \mathrm{BH}\alpha_2^0\,]}{2\,(\mathrm{B}^2 - \mathrm{H}^2)\,(4\mathrm{B}^2 - \mathrm{H}^2)^2 - \mathrm{B}\varkappa^2\,(2\mathrm{B}^2 - \mathrm{H}^2)^2} \cdot \quad (95)$$

Proposition 7 Comparison between equilibrium quantities under Regimes D (distributor sells products 1 and 2 ; none advertises) and MD (distributor sells products 1 and 2 ; both firm 1 and the distributor advertise) —how firm 1 and distributor advertisements influence equilibrium quantities :

$$x_1^D < x_1^{MD},\ r_1^D < r_1^{MD},\ p_1^D < p_1^{MD},\ \textstyle\prod_1^D < \textstyle\prod_1^{MD}$$

$$\gamma > 0,\, (\beta - \gamma)\, m K_D - \gamma\varkappa K_1 \geq 0 \Rightarrow x_2^D < x_2^{MD},\ r_2^D < r_2^{MD},\ p_2^D < p_2^{MD},\ \textstyle\prod_2^D < \textstyle\prod_2^{MD}$$

$$\gamma > 0,\, (\beta - \gamma)\, m K_D - \gamma\varkappa K_1 < 0 \Rightarrow x_2^D > x_2^{MD},\ r_2^D > r_2^{MD},\ p_2^D > p_2^{MD},\ \textstyle\prod_2^D > \textstyle\prod_2^{MD}$$

$$\gamma < 0 \Rightarrow x_2^D < x_2^{MD},\ r_2^D < r_2^{MD},\ p_2^D < p_2^{MD},\ \textstyle\prod_2^D < \textstyle\prod_2^{MD}$$

The distributor's advertisement also increases the demand for product 2. Therefore, firm 1's advertisement, which informs consumers about the existence, price, and characteristics of the product, has spillover effects. However, proposition 7 indicates that firm 1's equilibrium output, price, wholesale price, and profit increases in spite of the spillover effects from advertising. As regards the substitutes of the advertised product, the more the distributor advertises, the higher the equilibrium output, price, wholesale price, and profit. On the contrary, with only nominal advertisement by the distributor, the substitution effect would be greater than the advertising effect, and the equilibrium output, price, wholesale price, and profit would decrease.

Proposition 8 Comparison between equilibrium quantities under Regimes M (distributor sells products 1 and 2 ; only firm 1 advertises product 1) and MD (distributor sells products 1 and 2 ; both firm 1 and distributor advertise) —how distributor advertisements change the equilibrium :

$$x_1^{MD} < x_1^M,\ r_1^{MD} < r_1^M,\ p_1^{MD} < p_1^M,\ \textstyle\prod_1^{MD} < \textstyle\prod_1^M$$

$$\gamma < 0 \Rightarrow x_2^{MD} < x_2^M,\ r_2^{MD} < r_2^M,\ p_2^{MD} < p_2^M,\ \textstyle\prod_2^{MD} < \textstyle\prod_2^M$$

$$\gamma \geq 0 \Rightarrow x_2^{MD} \geq x_2^M,\ r_2^{MD} \geq r_2^M,\ p_2^{MD} \geq p_2^M,\ \textstyle\prod_2^{MD} \geq \textstyle\prod_2^M$$

Proposition 8 indicates that distributor advertising increases the demand for product 2 but firm 1 advertising decreases the demand for product 2, leading to a drop in the equilibrium output, price, wholesale price, and profit of product 2. As regards the substitutes of the advertised product, if distributor advertising is limited, their demand decreases from the substitution effect.

4. Conclusions

Advertising increases not only product recognition but also firm recognition. Advertisements inform consumers about the ordinary characteristics of products, creating spillover effects that increase the demand for substitutes of the advertised product. However, advertising unique product characteristics do not lead to spillover effects. Therefore, advertisements with little spillover effects increase profits and decrease the demand for substitutes of the advertised product.

Distributors advertise products of various producers, increasing the demand for unadvertised products from the spillover effects of advertising. Therefore, if distributor advertisements have large spillover effects,

firms producing substitutes can increase their profits by following a no−advertisement strategy. For example, we hardly see advertisements of small−size household electrical appliance manufacturers Yamazen and Twin Bird, but large−size distributors sell many electronics devices of small−size manufacturers.

In this paper, we analyzed the spillover effects of advertisements as well as the effects of distributor advertisements. From proposition 1, when firm 1 advertises without spillover effects, the equilibrium output, price, and profit decrease for substitutes but increase for complements. Proposition 2 indicates that an advertising campaign increases the demand for the advertised product even if it has spillover effects. Advertisements with a high rate of spillover increases the demand for both complements and substitutes of the advertised product. Proposition 3 indicates that the increase in the equilibrium output, price, and profit of the advertised product is greater without spillover effects than with such effects. Because of spillover effects, the advertising campaign increases the demand for the product of the rival firm, with the decrease in the demand−increasing effects of own−product advertisements. Proposition 4 indicates that the advertising campaign for product 1 increases its consumer surplus. The less the spillover effects of advertising, the greater is the consumer surplus of the advertised product. With a high spillover rate of advertising, advertising campaigns can increase the consumer surplus of the substitutes for the advertised product. Proposition 5 indicates that advertising campaigns decreases the profit from the substitutes of the advertised product. However, the advertising campaign can increase the rival's profit with a high spillover rate of advertising. Thus, with a high spillover rate of advertising, advertising campaigns without distributors can increase not only the consumer surplus and producer surplus of the advertised product but also the consumer surplus and producer surplus of substitutes for the advertised product. Proposition 6 indicates that when a distributor sells products 1 and 2, the equilibrium output, price, and profit increases for product 1, from the advertising campaign of firm 1, but decreases for substitutes. Proposition 7 indicates that firm 1's equilibrium output, price, wholesale price, and profit increases in spite of the spillover effects from advertising. As regards the substitutes of the advertised product, the more the distributor advertises, the higher are the equilibrium output, price, wholesale price, and profit. Proposition 8 indicates that, when both firm 1 and the distributor advertise, the demand of the substitutes of the advertised product decreases from the substitution effect if the distributor advertising is limited. Therefore, manufacturers of substitutes of wholesale products marketed through distributors can profit without advertising. These results explain why we hardly see advertisements of small−size household electrical appliance manufacturers. We must note that changing the specification of the model may lead to different results. In a future study, we intend to analyze the uncertainty of advertisements and examine how they influence repeat purchases.

Appendix

The proof of proposition 4 :
In Figure 1, ΔACF and ΔBDE represent consumer surplus at x_2^S and x_2^N, respectively. Line AF is parallel to BE, and CF is parallel to DE, so that ΔACF is similar to ΔBDE. Therefore, if $x_2^S > x_2^N$, $CS_2^S > CS_2^N$.

Figure 1. Consumer surplus for product 2 with and without advertisement
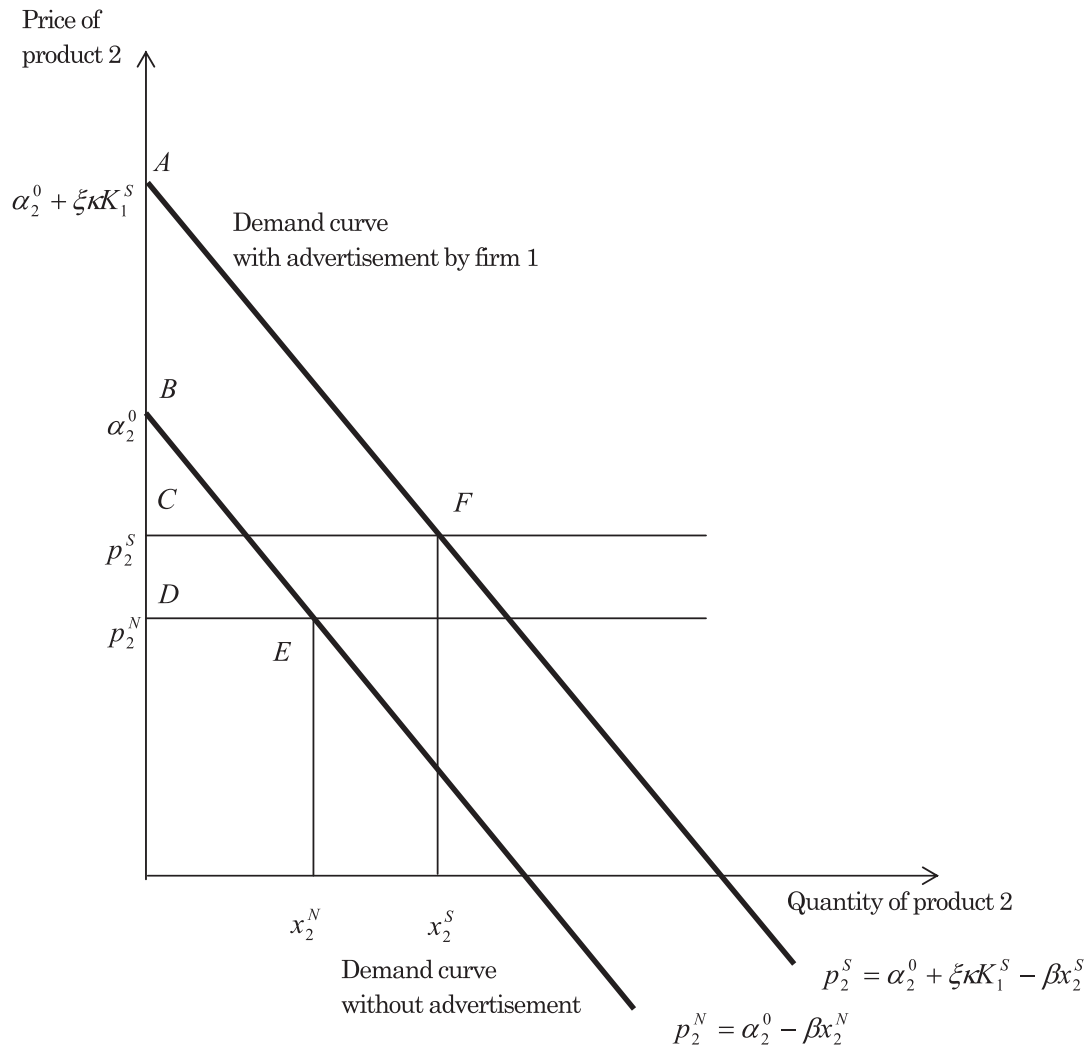
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An analysis of advertising campaigns
by distributors and manufacturers

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Today, many electrical appliances are sold by distributors. Therefore, consumers can obtain information on various products from the distributors' homepages on the Internet. Nonetheless, studies on how distributors influence manufacturers' advertising strategies are scarce. In this paper, we highlight the spillover effects of advertising and the impact of distributors' advertisements. The aim of this analysis is to clarify that distributors influence the advertising strategy of manufacturers. The results of the analysis indicate that some small manufacturers can earn profits without themselves advertising if many manufacturers produce similar goods.