# Advertising and Services of Retail Shops in Spatial Competition 

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## 1. Introduction

Consumers purchase various goods at retail shops. The demand behavior of consumers is constrained by shopping trip costs when these shops are separated spatially from rivals. Shopping trip costs give the spatial firm some monopoly power over customers in close proximity to the shop. Thus, the spatial firm can enlarge its market area by attracting consumers on the borderline of two markets. Then, advertising is one strategy to expand its market share.

Some advertising results in spillover effects, which are present when the advertising of one firm increases the amount sold without advertising. Generally, the products that retail shops sell are homogeneous. Therefore, the advertising of these shops causes spillover effects. If such effects are present, each retailer would have incentive to become a free rider, that is, receive the benefits of advertising without the costs. If there were a way to differentiate retail shops, their advertising would not cause spillover effects. We consider that theirservices differentiate these shops. For example, an electric appliance store replaces lamp bulbs for senior customers; fishmongers cook fish in ways that customers prefer (boiled, grilled, steamed, etc.) ; and bookstores give customers the information they seek about books. When retail shops supply these services, advertising of their services does not cause spillover effects.

The purpose of this study is to analyze how retail shop advertising affects market share in spatial competition and what actions such shops can take to be more competitive than their rivals. The rest of the paper is structured as follows. Section 2 presents a simple survey of previous approaches to such research. Section 3 builds the basic model. Section 4 analyzes the effects of advertising. Section 5 concludes and discusses how retail shops can become more competitive than their rivals.

## 2. Previous Research Approaches

Dorfman and Steiner (1954) are famous for their analysis of advertising. They prove that advertising is useful to differentiate own products from the similar products of rivals. Nelson (1974) analyzes the relationship between social awareness and repeat purchases and assumes that advertising expands initial sales for all brands but increases repeat purchases of superior brands. However, he does not provide a complete model of market competition. Kihlstrom and Riordan (1984) develop two models of advertising as a signal using Nelson's (1974) argument. They prove that superior brands acquire profit from advertising more than inferior brands and that advertising can signal quality. Roberts and Samuelson (1988, p.201) note that "advertising affects primarily the level of market demand rather than the distribution of market shares." By contrast, Kelton and Kelton (1982) show that advertising leads primarily to a shift from one brand to another. Bloch and Manceau (1999) analyze the effect of persuasive advertising between two competing products by using a model in which consumers differ in their tastes. Bloch and Manceau (1999) show that "advertising may induce a decrease in the price of the advertised product, showing that a firm does not necessarily have an incentive to engage in the advertising" strategy of manufactures. Aoba (1997) analyzes the interaction of advertising and advertising intensity and proves that advertising increases awareness of
firms and targeted advertising increases profit. Aoba (1998) compares advertising by manufacturers with advertising by retailers and proves that advertising expenditure is added to wholesale prices and advertising decreases the consumer surplus. Aoba (1998) does not consider that advertising gives consumers information about products, product prices, and brands. The benefits of such information could alter Aoba's (1998) conclusions. Aoba (2014) analyzes how distributors influence the advertising strategy of manufacturers, proving that advertising campaigns decreases profit from substitutes of the advertised product and that some small manufacturers can earn profits without themselves advertising if many manufacturers produce similar goods. However, Aoba (2014) does not consider that advertising of retail shops differentiate these shops. Mathewson-Winter (1986) demonstrates that spillover effects decrease the advertising of retailers and vertical restraints are formed to avoid free riding. When retail shops supply original services, advertising of their services does not cause spill over effects. We consider that advertising of retail shop services differentiates these shops. We analyze how retail shop advertising affects market share in spatial markets. Nakagome (1989, p.54) "demonstrates the neutrality effects of autonomous demand on retailers' location as well as on their stable market up pricing" in the spatial context. Retail shops often have "countervailing power" when manufacturers have monopoly power. Nakagome (1990) analyzes how 'countervailing power' influences the retail price and the market share, proving that increasing countervailing power decreases the wholesale price and does not influence the retail price and market share. Using Nakagome's model, we analyze the advertising of retail shops in spatial competition.

## 3. Basic Model

Following the standard spatial model, we consider a linear spatial economy of length 1. As we see from Figure 1, retailer 0 is located at the endpoint 0 of the line segment, and retailer 1 is located at the other endpoint 1 . For simplicity, consumers are uniformly distributed along the market line $[0,1]$. The goods retailers sell are supposed to be homogeneous; we define $x_{0}$ to be the demand of retailer 0 and $x_{1}$ to be the demand of retailer 1 . We define $p_{0}$ as the price of $x_{0}$ and $p_{1}$ as the price of $x_{1}$. A representative consumer is spatially separated from a firm by $r$ miles, while $t$ is the monetary shopping trip cost per mile and is assumed to be constant.

## Location of retail shop 0

## Location of retail shop 1



Figure 1. Locations of retail shops and consumer
First, we consider the utility-maximizing behavior of consumers. Let $U$ denote the utility function of a representative consumer. For simplicity, we define $m$ as all consumption other than $x_{0}, x_{1}$. We define that the unit price of $m$ as equals to 1 . The utility of the consumer, $U$, is an increasing function of individual consumption $x_{i}(i=0,1)$ and $m$. The consumer purchases $x_{i}$ when the utility of $x_{i}$ is larger than $x_{j}$ ( $i \neq$ $j$ ). For analytical convenience, the utility function $U_{i}\left(x_{i}, m\right)$ is specified as a Cobb-Douglas utility function.

$$
\begin{equation*}
U_{i}\left(x_{i}, m\right)=x_{i}{ }^{a} m^{b} \quad(i=0,1) \tag{1}
\end{equation*}
$$

The parameters $a$ and $b$ are constant and positive. To avoid confusion, we define $m_{i}(i=0,1)$ as $m$ when the consumer purchases $x_{i}(i=0,1)$.

The consumer determines the utility-maximizing quantity of $x_{i}$ and $m$, considering the following budget constraint.

$$
\begin{equation*}
Y-s-t r=p_{i} x_{i}+m_{i} \quad(i=0,1) \tag{2}
\end{equation*}
$$

In equation (2), $Y$ is the income earned by the consumer, and $t$ is the monetary shopping trip cost per mile. We assume that the shopping cost is constant, because the consumer purchases $x_{i}$ and $m_{i}$ on one occasion and his shopping cost does not depend on the quantity of his purchase. We assume that the consumer does not have information about prices. He has to search for price information at cost $s$, which is assumed constant. Equation (2) shows that the total amount of money spent on $x_{i}$ and $m_{i}$ equals his income $Y$ minus shopping costs $t r$ minus search cost $s$.

The consumer chooses the quantity of his purchase that maximizes utility function (1) subject to budget constraint (2).

$$
\begin{gather*}
x_{0}=\frac{a}{a+b} \frac{Y-s-t r}{p_{0}}  \tag{3}\\
x_{1}=\frac{a}{a+b} \frac{Y-s-t(1-r)}{p_{1}}  \tag{4}\\
m_{0}=\frac{a}{a+b}(Y-s-t r)  \tag{5}\\
m_{1}=\frac{a}{a+b}[Y-s-t(1-r)]  \tag{6}\\
\frac{\partial U}{\partial r}=\frac{-t(a+b)}{p_{0}}\left(\frac{a}{a+b}\right)^{a}\left(\frac{b}{a+b}\right)^{b}(Y-s-t r)^{a+b-1}<0  \tag{7}\\
\frac{\partial^{2} U}{\partial r^{2}}=\frac{t^{2}(a+b)(a+b-1)}{p_{0}}\left(\frac{a}{a+b}\right)^{a}\left(\frac{b}{a+b}\right)^{b}(Y-s-t r)^{a+b-2}  \tag{8}\\
a+b \geq 1 \Rightarrow \frac{\partial^{2} U}{\partial r^{2}} \geq 0 \\
a+b<1 \Rightarrow \frac{\partial^{2} U}{\partial r^{2}}<0
\end{gather*}
$$

Generally, utility decreases as distance to the retail shop lengthens and the marginal utility of the distance increases. Then, we obtain $a+b>1$ because of $\partial^{2} U / \partial r^{2}>0$.

Figure 2 shows that $U_{0}$ and $U_{1}$ decrease with the assumption of the position of consumer $(r)$ when $p_{0}=\bar{p}_{0}$ (constant) and $p_{1}=\bar{p}_{1}$ (constant).
$r^{*}$ is assumed as the location at which $U_{0}=U_{1}$. That is, when the consumer who is separated from retail shop 0 by $r^{*}$ miles purchases $x_{0}$, his utility is the same as when he purchases $x_{1}$. From (3), (4), (5), and (6),


Figure 2. Relationship between shopping trip cost and utility

$$
\begin{align*}
r^{*}= & \frac{(Y-s)\left(p_{1}{ }^{\alpha}-p_{0}{ }^{\alpha}\right)+t p_{0}{ }^{\alpha}}{t\left(p_{0}{ }^{\alpha}+p^{\alpha}\right)}  \tag{9}\\
& \text { where } \alpha=a /(a+b) .
\end{align*}
$$

From the utility-maximizing assumption,
If $r \leq r^{*}$, then the consumer purchases $x_{0}\left(x_{0}>0\right)$.
If $r>r^{*}$, then the consumer purchases $x_{1}\left(x_{1}>0\right)$.
When we define $q_{0}\left(q_{1}\right)$ as the demand that retail shop 0 (1) faces, $q_{0}\left(q_{1}\right)$ is written as follows:

$$
\begin{gather*}
q_{0}=n \int_{0}^{r^{*}} x_{0} d r=\frac{a}{a+b} \frac{n r^{*}\left(Y-s-\frac{t r^{*}}{2}\right)}{p_{0}}  \tag{10}\\
q_{1}=n \int_{r^{*}}^{0} x_{1} d r=\frac{a}{a+b} \frac{n\left(1-r^{*}\right)\left[Y-s-\frac{t\left(1-r^{*}\right)}{2}\right]}{p_{1}} \tag{11}
\end{gather*}
$$

In equations (10) and (11), $n$ is the density of consumers who are distributed uniformly along the market line[0, 1].
We define $c_{0}\left(c_{1}\right)$ as the cost of distribution by retailer 0 (1) and assume that the cost is only marginal cost and constant. Then, the profit of retailer 0 (1) is written as follows :

$$
\begin{gather*}
\Pi_{0}=\left(p_{0}-c_{0}\right) n \int_{0}^{r^{*}} x_{0} d r=\frac{a}{a+b} \frac{p_{0}-c_{0}}{p_{0}} n r^{*}\left(Y-s-\frac{t r^{*}}{2}\right)  \tag{12}\\
\Pi_{1}=\left(p_{1}-c_{1}\right) n \int_{t^{*}}^{1} x_{1} d r=\frac{a}{a+b} \frac{p_{1}-c_{1}}{p_{1}} n\left(1-r^{*}\right)\left[Y-s-\frac{t\left(1-r^{*}\right)}{2}\right] \tag{13}
\end{gather*}
$$

Market frontier location $r^{*}$ depends on retail prices.
From the first-order condition,

$$
\begin{equation*}
r^{*}\left(Y-s-\frac{t}{2} r^{*}\right) \frac{c_{0}}{p_{0}}+\left(p_{0}-c_{0}\right)\left(Y-s-t r^{*}\right) \frac{d r^{*}}{d p_{0}}=0 \tag{14}
\end{equation*}
$$

Generally, the market area of retailer 0 shrinks as $p_{0}$ rises because rising $p_{0}$ comparatively reduces the price of the rival retailer $\left(p_{1}\right)$ and the market area of retailer 1 expands. Then, retailer 0 cannot anticipate all reactions of retailer 1 and consumers, and so, $d r^{*} / d p_{0}$ is the predicted value. Following Nakagome (1990), we assume this value to be given.

$$
\begin{equation*}
\frac{d r^{*}}{d p_{0}}=-Z \tag{15}
\end{equation*}
$$

From (14) and (15), we obtain

$$
\begin{equation*}
r^{*}\left(Y-s-\frac{t}{2} r^{*}\right) \frac{c_{0}}{p_{0}}-z\left(p_{0}-c_{0}\right)\left(Y-s-t r^{*}\right)=0 \tag{16}
\end{equation*}
$$

The total differential of (16) yields

$$
\begin{equation*}
\frac{d p_{0}}{d r^{*}}=\frac{c_{0}-\left(Y-s-t r^{*}\right) p_{0}+t z\left(p_{0}-c_{0}\right) p_{0}{ }^{2}}{c_{0} r^{*}\left(Y-s-\frac{t}{2} r^{*}\right)+z\left(Y-s-t r^{*}\right) p_{0}{ }^{2}}>0 \tag{17}
\end{equation*}
$$

Because the sum of search costs and shopping trip costs is smaller than income, $Y-s-t r^{*}>0$. From this assumption, $p_{0}-c_{0}>0$. Consequently, (17) is positive. Equation (17) means that the price of the retail shop 0 increases with the retailer's market area. Equation (17) implies that the profit-maximizing condition (16) is upward sloping in Figure 2.

The total differential of (9) yields

$$
\begin{equation*}
\frac{d r^{*}}{d p_{0}}=-\frac{\frac{2 a}{a+b} p_{0}^{\frac{a}{a+b}-1} p_{1}^{\frac{a}{a+b}}\left(Y-s-\frac{t}{2}\right)}{t\left(p_{0}^{\frac{a}{a+b}}+p_{1}^{\frac{a}{a+b}}\right)^{2}}<0 \tag{18}
\end{equation*}
$$

Because the sum of search costs and shopping trip costs per mile is smaller than income, $Y-s-(t / 2)>0$. Consequently, the sign of (18) is negative. (18) means that raising the retail price shrinks his market area.

Equation (18) implies that the zero-profit condition (9) is downward sloping in Figure 2.
For retailer 1,

$$
\begin{equation*}
\frac{d p_{1}}{d r^{*}}=-\frac{c_{1}\left[Y-s-t\left(1-r^{*}\right)\right] p_{1}+t z\left(p_{1}-c_{1}\right) p_{1}{ }^{2}}{c_{1}\left(1-r^{*}\right)\left[Y-s-\frac{t}{2}\left(1-r^{*}\right)\right]+z\left[Y-s-t\left(1-r^{*}\right)\right] p_{1}{ }^{2}}<0 \tag{17-2}
\end{equation*}
$$

(17-2) means that the retail price decreases as his market area shrinks.

## 4. Effects of advertising

Advertising affects market share, retail prices, and retail profits. We examine the effects of advertising using the model derived in Section 3.

There are many varieties of advertising, for example, advertising that provides information about only quality or price, advertising that establishes brand value, and advertising that gives rise to popular trends. Advertising that introduces new services or novel ideas can inflict damage on rivals and increase the advertiser's own profit. In the United States, comparative advertising is used often to show the superiority of the advertiser's products. In this section, we examine how advertising expands market area.

### 4.1. Advertising to differentiate

Some advertising is designed to differentiate. For example, in the case of advertising to establish brand value, there is little spillover of advertising effects. We do not consider that retail shop advertising can establish brand value. However, if these shops advertise original services, they could increase demand without spillover effects. For example, only consumers that purchase goods in retail shop 0 can obtain such services as after-sales care. We assume that each retail shop can shift its utility function upward by advertising at the first stage. We define $K_{0}$ as the advertising costs of retailer $0, K_{1}$ as the advertising costs of retailer $1, a_{0}\left(K_{0}\right)$ as the parameter of the utility function of $x_{0}, U_{0}$ and $a_{1}\left(K_{1}\right)$ as the parameters of the utility function of $x_{1}, U_{1}$. We assume $d a_{0} / d K_{0}>0, d a_{1} / d K_{1}>0$. That is $x_{0}$, and $x_{1}$ are homogeneous, but advertising to differentiate increases utility: advertising by retailer 0 (1) increases the utility of $x_{0}\left(x_{1}\right)$.We reformulate the utility function to involve these advertising effects.

$$
\begin{align*}
& U_{0}\left(x_{0}, m_{0}\right)=x_{0}{ }^{a_{0}\left(K_{0}\right)} m_{0}{ }^{b}  \tag{19}\\
& U_{1}\left(x_{1}, m_{1}\right)=x_{1}{ }^{a_{1}\left(K_{1}\right)} m_{1}{ }^{b} \tag{20}
\end{align*}
$$

From (19),

$$
\begin{gather*}
\log U_{0}\left(x_{0}, m_{0}\right)=a_{0}\left(K_{0}\right) \log x_{0}+b \log m_{0}  \tag{21}\\
\frac{\partial \log U_{0}}{\partial K_{0}}=\frac{\partial a_{0}}{\partial K_{0}} \log x_{0}+a_{0}\left(K_{0}\right) \frac{1}{x_{0}} \frac{\partial x_{0}}{\partial K_{0}}+\frac{b}{m_{0}} \frac{\partial m_{0}}{\partial K_{0}}=\frac{\partial a_{0}}{\partial K_{0}} \log x_{0}>0 \tag{22}
\end{gather*}
$$



Figure 3. Subgame Nash perfect equilibrium with advertising for differentiation

The sign of (22) holds because an increase of $x_{0}$ by the advertising of retailer 0 decreases all consumption but $x_{0}: a_{0}\left(K_{0}\right)\left(1 / x_{0}\right)\left(\partial x_{0} / \partial K_{0}\right)>(b / m)\left(\partial m_{0} / \partial K_{0}\right)$. Figure 3 shows that advertising without spillover effects by retailer 0 shifts $U_{0}$ upward.

If advertising of $x_{0}$ increases the demand $x_{0}$ of and there is little spillover of advertising effects, then the advertising of $x_{0}$ expands its market share, $d r^{*} / d K_{0}>0$ (Figure 4).

$$
\begin{equation*}
\frac{\partial q_{0}}{\partial K_{0}}=\frac{b}{(a+b)^{2}} \frac{n r^{*}\left(Y-s-\frac{t r^{*}}{2}\right)}{p_{0}} \frac{\partial a_{0}}{\partial K_{0}}+\frac{a}{a+b} \frac{n\left(Y-s-t r^{*}\right)}{p_{0}} \frac{\partial r^{*}}{\partial K_{0}}>0 \tag{23}
\end{equation*}
$$

In the second stage, each retailer determines the profit-maximizing quantity of the retail price considering the demand represented by (10) and (11).Then, in Figure 2, subgame Nash perfect equilibrium is $E$. We define $p_{0}^{*}$ and $p_{1}^{*}$ as equilibrium prices. The maximization problem that retail shop 0 faces in the first stage is

$$
\begin{equation*}
\max . \Pi_{0}=\left(p_{0}{ }^{*}\left(K_{0}\right)-c_{0}\right) q_{0}\left(K_{0}\right)-K_{0} . \tag{24}
\end{equation*}
$$

We obtain the first-order condition that must hold at the optimal advertising expenditure, $K^{*}$.

$$
\begin{array}{r}
\frac{d \Pi_{0}}{d K_{0}}=\left(p_{0}^{*}-c_{0}\right) \frac{\partial q_{0}}{\partial K_{0}}-1+q_{0} * \frac{\partial p_{0} *}{\partial r^{*}} \frac{\partial r^{*}}{\partial K_{0}}=0  \tag{25}\\
(+)
\end{array}
$$

From (15), $\partial p_{0}{ }^{*} / \partial r^{*}>0$ holds. The first and second terms on the right-hand side are the direct effects of advertising. The third term on the right-hand side implies that market expansion by advertising increases the retailer's profit.

The advertising effects on the retailer's rival are as follows.

$$
\begin{gather*}
\frac{\partial q_{1}}{\partial K_{0}}=\frac{a}{a+b} \frac{n\left[Y-s-t\left(1-r^{*}\right) r^{*}\right]}{p_{1}} \frac{\partial r^{*}}{\partial K_{0}}<0  \tag{26}\\
\frac{d \Pi_{1}}{d K_{0}}=\left(p_{1}^{*}-c_{1}\right) \frac{\partial q_{1}^{*}}{\partial K_{0}}+q_{1}{ }^{*} \frac{\partial p_{1}^{*}}{\partial r^{*}} \frac{\partial r^{*}}{\partial K_{0}}<0  \tag{27}\\
(-) \quad(-)(+)
\end{gather*}
$$

If the advertising of a retail shop allows its customers to identify its original service, then there is little spillover effect. In this case, advertising expands the market area and increases the retailer's profit. On the other hand, advertising shrinks the market share of the rival and decreases its profit.

### 4.2. Price advertising

Price advertising is a way for consumers to obtain information about the prices of goods. If there is no price advertising, then consumers have to search for prices by themselves. Consequently, price advertis-


Figure 4. The effect of advertising (without spillover effects)
ing decreases search costs.
Now, we assume that advertising by each retail shop can decrease consumers' search costs in the first stage. That is, consumers' search costs are a decreasing function of retailers' advertising, $s=s(K), d s / d K$ $<0$. From (9),

$$
\begin{align*}
\frac{\partial r^{*}}{\partial s}= & -\frac{p_{1}{ }^{\alpha}-p_{0}{ }^{\alpha}}{t\left(p_{0}{ }^{\alpha}+p_{1}{ }^{\alpha}\right)}  \tag{28}\\
& \text { where } \alpha=a /(a+b) .
\end{align*}
$$

Therefore, we obtain

$$
\begin{align*}
& p_{1}>p_{0} \Rightarrow \frac{\partial r^{*}}{\partial s}<0 \Rightarrow \frac{\partial r^{*}}{\partial K_{0}}>0  \tag{29}\\
& p_{1} \leq p_{0} \Rightarrow \frac{\partial r^{*}}{\partial s}>0 \Rightarrow \frac{\partial r^{*}}{\partial K_{0}} \leq 0 \tag{30}
\end{align*}
$$

From (29) and (30), price advertising expands the market share of the lower priced retail shop because price advertising uniformly decreases customers' search costs. Consumers that are interested in advertised goods seek the lowest prices, which are not necessarily available at the advertised retail shops.

From (10),

$$
\begin{gather*}
\frac{\partial q_{0}}{\partial K_{0}}=\frac{a}{a+b} \frac{n}{p_{0}}\left[\left(Y-s-t r^{*}\right) \frac{\partial r^{*}}{\partial K_{0}}-r^{*} \frac{d s}{d K_{0}}\right]  \tag{31}\\
(-) \\
p_{1}>p_{0} \Rightarrow \frac{\partial r^{*}}{\partial K_{0}}>0 \Rightarrow \frac{\partial q_{0}}{\partial K_{0}}>0  \tag{32}\\
p_{1} \leq p_{0} \Rightarrow \frac{\partial r^{*}}{\partial K_{0}} \leq 0,\left|\left(Y-s-t r^{*}\right) \frac{\partial r^{*}}{\partial K_{0}}\right| \geq\left|r^{*} \frac{d s}{d K_{0}}\right| \Rightarrow \frac{\partial q_{0}}{\partial K_{0}} \leq 0  \tag{33}\\
p_{1} \leq p_{0} \Rightarrow \frac{\partial r^{*}}{\partial K_{0}} \leq 0,\left|\left(Y-s-t r^{*}\right) \frac{\partial r^{*}}{\partial K_{0}}\right|<\left|r^{*} \frac{d s}{d K_{0}}\right| \Rightarrow \frac{\partial q_{0}}{\partial K_{0}}>0 \tag{34}
\end{gather*}
$$

From (32), (33), and (34), price advertising by retail shop 0 increases its demand when its retail price is lower than the price of the rival shop. Price advertising by retail shop 0 increases the demand of the rival shop when the retail price of the former is higher than the price of the latter. Then, price advertising by retail shop 0 sometimes increases its demand, but sometimes decreases it.
In the second stage, each retailer determines the profit-maximizing quantity of the retail price considering demand functions (10) and (11). Then, subgame Nash perfect equilibrium is $E$ in Figure 2. We define $p_{0}{ }^{*}$ and $p_{1}{ }^{*}$ as equilibrium prices. The maximization problem that retail shop 0 faces in the first stage is

$$
\begin{equation*}
\max . \Pi_{0}=\left(p_{0}{ }^{*}\left(K_{0}\right)-c_{0}\right) q_{0}\left(K_{0}\right)-K_{0} \tag{35}
\end{equation*}
$$

We obtain the first-order condition that must hold at the optimal advertising expenditure, $K_{0}{ }^{*}$.

$$
\begin{equation*}
\frac{d \Pi_{0}}{d K_{0}}=\left(p_{0}^{*}-c_{0}\right) \frac{\partial q_{0}{ }^{*}}{\partial K_{0}}-1+q_{0}^{*} \frac{\partial p_{0}{ }^{*}}{\partial K_{0}}=0 \tag{36}
\end{equation*}
$$

The first and second terms on the right-hand side imply the direct effect of advertising. The third term on the right-hand side implies that market expansion by advertising increases the retailer's profit. The sign of $\left(\partial q_{0}{ }^{*} / \partial K_{0}{ }^{*}\right)$ depends on the price of the rival shop. From (17), we obtain $\partial q_{0} / \partial r^{*}>0$. Therefore, the influence that price advertising of retail shop 0 has on $p_{0}$ is as follows.

$$
\begin{array}{r}
p_{1}>p_{0} \Rightarrow \frac{\partial r_{0}^{*}}{\partial K_{0}}=\frac{\partial p_{0}{ }^{*} \frac{\partial r^{*}}{\partial r^{*}} \frac{\partial s}{\partial s} \frac{\partial S}{\partial K_{0}}>0}{} \begin{array}{r}
(+)(-)(-) \\
p_{1} \leq p_{0} \Rightarrow \frac{\partial r_{0}^{*}}{\partial K_{0}}=\frac{\partial p_{0}^{*} * \frac{\partial r^{*}}{\partial r^{*}} \frac{\partial s}{\partial s} \leq 0}{\partial K_{0}} \leq 0 \\
(+)(+)(-)
\end{array}
\end{array}
$$

From (37) and (38), in the case of price advertising, both the direct and price effects depend on relative
price. Price advertising by retail shop 0 expands its market share and equilibrium price $p_{0}{ }^{*}$ rises when the retail price of retail shop 0 is lower than the price of the rival shop. Then, price advertising shifts the profit-maximization curve BB upward in Figure 5

The influence that the price advertising of retail shop 0 has on the demand and price of rival shop 1 is as follows.

$$
\begin{gather*}
p_{1}>p_{0},\left[Y-s-t\left(1-r^{*}\right)\right] \frac{\partial r^{*}}{\partial s}+\left(1-r^{*}\right) \geq 0 \Rightarrow \frac{\partial q_{1}}{\partial K_{0}} \geq 0  \tag{39}\\
p_{1}>p_{0},\left[Y-s-t\left(1-r^{*}\right)\right] \frac{\partial r^{*}}{\partial s}+\left(1-r^{*}\right)<0 \Rightarrow \frac{\partial q_{1}}{\partial K_{0}}<0  \tag{40}\\
p_{1}>p_{0} \Rightarrow \frac{\partial q_{1}}{\partial K_{0}}>0
\end{gather*}
$$

From (9), we have

$$
\begin{gather*}
\frac{\partial r^{*}}{\partial p_{1}}=\frac{2 \alpha p_{0}{ }^{\alpha} p_{1}{ }^{\alpha-1}\left(Y-s-\frac{t}{2}\right)}{t\left(p_{0}{ }^{\alpha}+p_{1}{ }^{\alpha}\right)^{2}} \quad \text { where } \alpha=\frac{a}{a+b}  \tag{42}\\
\frac{\partial r^{*}}{\partial p_{1}}=-\frac{p_{0}}{p_{1}} \frac{\partial r^{*}}{\partial p_{0}} \tag{43}
\end{gather*}
$$

From (13), we obtain

$$
\begin{equation*}
r^{*}\left(Y-s-\frac{t r^{*}}{2}\right) \frac{c_{1}}{p_{1}}+\left(p_{1}-\mathrm{c}_{1}\right)\left(Y-s-t r^{*}\right) \frac{\partial r^{*}}{\partial p_{1}}=0 \tag{44}
\end{equation*}
$$

From (14), (43), and (44), the following equations hold in equilibrium.

$$
\begin{gather*}
p_{1}^{*}=\frac{c_{1}}{c_{0}}\left(2 c_{0}-p_{0}\right)  \tag{45}\\
\frac{\partial p_{1}^{*}}{\partial r^{*}}=-\frac{c_{1}}{c_{0}} \frac{\partial p_{0}^{*}}{\partial r^{*}}  \tag{46}\\
p_{1}>p_{0} \Rightarrow \frac{\partial p_{0}^{*}}{\partial K_{0}}>0, \frac{\partial p_{1}^{*}}{\partial K_{0}}<0  \tag{47}\\
p_{1} \leq p_{0} \Rightarrow \frac{\partial p_{0}^{*}}{\partial K_{0}} \leq 0, \frac{\partial p_{1}^{*}}{\partial K_{0}} \geq 0 \tag{48}
\end{gather*}
$$

From (47), price advertising of retail shop 0 raises its equilibrium price and reduces the equilibrium price of rival shop 1 when $p_{0}$ is smaller than $p_{1}$. From (48), price advertising of retail shop 0 raises the equilibrium price of rival shop 1 and reduces the equilibrium price of retail shop 0 when $p_{0}$ is higher than $p_{1}$.

The influence that the price advertising of retail shop 0 has on the profit of rival shop 1 is as fol-


Figure 5. The effect of advertising with spillover effects
lows.

$$
\begin{gather*}
\frac{\partial \Pi_{1}^{*}}{\partial K_{0}}=\left(p_{1}^{*}-c_{1}\right) \frac{\partial q_{1}^{*}}{\partial K_{0}}+q_{1}^{*} \frac{\partial p_{1}^{*}}{\partial K_{0}}  \tag{49}\\
p_{1}>p_{0},\left[Y-s-t\left(1-r^{*}\right)\right] \frac{\partial r^{*}}{\partial s}+\left(1-r^{*}\right)<0 \Rightarrow \frac{\partial \Pi_{1}^{*}}{\partial K_{0}}<0  \tag{50}\\
p_{1} \leq p_{0} \Rightarrow \frac{\partial \Pi_{1}^{*}}{\partial K_{0}}>0 \tag{51}
\end{gather*}
$$

From (50) and (51), price advertising of retail shop 0 increases the profit of rival shop 1 when retail price $p_{0}$ is higher than that of rival $p_{1}$. The reason advertising of a retail shop increases the profit of its rival is that price advertising uniformly decreases the search costs of consumers (the spillover effects of advertising). Thus, in order to avoid the spillover effects of advertising, shops need to advertise in order to differentiate. To return to a previous example, fishmongers need to prepare fish in ways preferred by customers.

## 5. Conclusions

In this study, we examine how retail shop advertising affects market share, prices, and profits in a spatial competition model. Therefore, price advertising and commodity advertising cause spillover effects of advertising. However, advertising to differentiate these shops does not cause spillover effects of advertising and expands the retailer's own market share. If a retail shop provides customers with original services and advertises its services, then that advertising does not cause spillover effects. We prove that advertising by these shops expands their market share when advertising does not cause spillover effects.
Shops at which consumers purchase goods are not only retail shops. Other outlets include delivery shops and the internet. Many items are sold both at retail shops and on the internet. Internet sites are convenient for 24 -hour purchasing. It is difficult for retail shops to out-compete other such shops and internet shopping. In this study, we consider the services that retailers offer as effective methods of competing against internet shopping. Some consumers purchase goods on the internet after actually looking at goods at retail shops because internet shopping prices are lower than those of these shops. However, if only the retail shop provides original service to customers, then it may not be deprived of customers. Retail shops need to devise original services in order to survive in the market. We leave how advertising affects the competition between these shops and internet shopping for future analysis.

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# Advertising and Services of Retail Shops in Spatial Competition 

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Consumers purchase various goods at retail shops. Because products sold at such shops are homogeneous, the advertising of these shops causes spillover effects and each retailer would have incentive to become a free rider. We consider that services offered by retail shops differentiate these shops, in which case, advertising of their services would not have spillover effects. We analyze the advertising of retail shops in spatial competition and prove that advertising of their services expands market share when these shops supply original services. Moreover, we consider the services that retailers offer as effective methods of competing against internet shopping.

