

New Approach to the Concept-Based Teaching and Learning Mathematics at Primary School Level in Papua New Guinea: Concrete-Representational-Abstract (CRA) Sequence of Instruction Guided through Structured Japanese Lesson Pattern

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Abstract

Papua New Guinea (PNG) abolished Outcome Based Education (OBE) curriculum as same as the other countries including South Africa (SA). PNG is now under the transition to Standard Based Education (SBE) as a new educational approach. Its primary intention is to improve the performance of students with standards of important subject like mathematics. In this context, it would be crucial to seek effective and applicable methods for teaching and learning of mathematics primarily focusing on the conceptual understanding articulated well with practical skills at the primary school level. Thus, we investigate the applicability of Concrete-Representational-Abstract (CRA) sequence of instruction guided through Japanese lesson pattern to the teaching and learning of the comparison of fraction size. CRA sequence of instruction provides research-based practical framework for lesson design and has been proven to improve the students' conceptual understanding of mathematics. The Japanese lesson pattern as a structured 'problem solving' approach has been identified as one of effective practical framework for mathematics lesson delivery in the Trends in International Mathematics and Science Study (TIMSS) video tape study. In this paper, firstly we indicate quantitatively and qualitatively that our proposed approach, CRA sequence of instruction guided through structured problem-solving Japanese lesson pattern, has appreciable impact to raise the conceptual understanding of fraction in mathematics education at the primary school level in PNG. The implication of our study is also discussed for the improvement of mathematics education in the educational context of PNG.

Keywords : Papua New Guinea, Fraction magnitude, Concrete-Representational-Abstract (CRA) Sequence, Structured problem-solving Japanese lesson pattern, Misconception analysis.

1. Introduction

Over the last two decades several educational reforms have been made to improve the quality of education in PNG. In 2001, the Outcome Based Education (OBE) approach was introduced to support the implementation of the primary school curriculum

(Grade3-8) with a focus on learning in the cultural relevance and development of lifelong skills (National Department of Education, 2003; Neofa, 2010). However, OBE implementation was greatly challenging in PNG because of an inadequate resource support, low extent of professional development for teachers, absence of national curriculum framework and over-

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reliance on implementation by Western consultants with irrelevant contents to PNG cultural context (Agigo, 2010; Neofa, 2010). In 2013, the Department of Education (NDoE) has abolished OBE curriculum and made shift into adopting a new educational approach called as Standard Based Education (SBE) according to recommendation through Czuba (2013) report from the OBE Task Force (OTF) assigned by NDoE.

NDoE (2015) in PNG defines “standard” as a level of quality or achievement, especially the level that is thought to be acceptable to keep students’ performance at a preferable level. With the focus to achieve the intention of SBE curriculum in a classroom context, the Japan International Cooperation Agency (JICA) funded the “Quality in Science and Mathematics Education (QUISME)” project through assisting PNG Curriculum Development Division (CDD) from 2014. The QUISME project is aiming ultimately at providing the specific standards that focus on raising conceptual understanding of students. Its primary approach is to incorporate practical activities to encourage their participation in a lesson as well as for the development of their conceptual understanding (Chandler, Forture, Lovett & Scherner; 2006). The urgent need to develop the students’ conceptual understanding has been strongly recommended in the recent study of Apule (2017) in the field of fraction at primary school level in PNG.

Siegler, Fazio, Bailey & Zhou (2012) emphasized on the support for teachers to employ the materials and methodologies to enhance the understanding of the mathematical concept of students. In our study, two types of research-based instructional approach has been examined to improve students’ conceptual understanding of mathematics. One is the “Concrete-Representational-Abstract (CRA)” sequence of instruction. Originally, its primary target is the students with difficulties or at risk in learning mathematics. Then its focus is to build up mathematical concept within such students by exposing them to a series of lessons sequentially from concrete to abstract through representational stages. Many works indicated the CRA sequence of instruction were effective in the improvement of conceptual understanding of mathematics of students with mathematics disabilities. These include the CRA application in the fraction instruction (Buttler, Miller, Crehan, Babbit and Pierce 2003), the algebraic

transformation equation (Witzel, Mercer and Miller 2003) and linear function (Witzel 2005) at a secondary level in USA. National Council of Teachers of Mathematics (NCTM) of USA also suggested that CRA could be beneficial to all students (Berkas & Pattison, 2007).

The other is structured ‘problem solving’ approach in the Japanese lesson pattern (Stigler & Hiebert, 1997). It is the instructional approach for mathematical lessons identified in TIMSS lesson study (Stigler, Gonzales, Kawanaka, Knoll & Serrano 1997). This Japanese standard approach of teaching mathematics lesson provides students with opportunities to develop their mathematical abilities including conceptual and procedural understanding (Shimizu, 2000). We think this approach was significant for our study as it makes possible for a teacher to design the lesson to create the interest in mathematics of students and stimulate creative mathematical activity in a classroom through collaborative work among them.

The primary purpose of this study is to test the applicability of the CRA sequences of instruction guided through Japanese lesson ‘pattern’ in teaching and learning of mathematics by contextualizing the materials and methods to PNG primary mathematics in the transition from the OBE to SBE. Thus, our research questions are as follows;

- Can CRA sequence of instruction guided through Japanese lesson pattern make any positive impact to the improvement of mathematical concept of students at a primary level of PNG?
- Has CRA sequence guided through Japanese lesson pattern dependency on the type of students’ misconceptions?

Approaching to the first research question, the target topic is the concept of fraction, since it is one of the most challenging area as well as a central concept to understand other topics according to many works on mathematics education (Lamon, 1999; Litwiller & Bright, 2002; Byrnes & Wasik, 1991; Siegler *et al.* 2012). Author (2017) also confirmed that fraction concept was an area in which many upper primary students in PNG had difficulty in their understanding or in getting a solution. Many researchers agreed that a greater emphasis on concepts, specifically size, should improve the efficacy of teaching fractions (Byrnes & Wasik 1991; Lamon 2007). The implication of our study

is also discussed in the current educational context in PNG in the transition from OBE to SBE.

2. Theoretical Background

2.1 CRA Sequence of Instruction

From the practical perspective, the CRA approach consists of three stages, specifically including concrete, representational and abstract stages in the progress of teaching and learning. Here we would like to explain CRA focusing primarily on the work of Witzel (2005).

The *concrete stage of understanding* is the most basic stage for mathematical understanding. It is also the most crucial stage for developing conceptual understanding of mathematical concepts/skills. Concrete learning occurs when students have ample opportunities to manipulate concrete objects to solve problems (Witzel, 2005). For students who have mathematical learning challenges, it needs for teacher to provide students with explicit model for the use of specific concrete object to solve specific problems.

The *representational stage of understanding* is the intermediate stage between concrete and abstract ones where students learn how to solve their problems by drawing pictures, illustrations or diagrams. These represent illustratively or diagrammatically the concrete objects that students manipulated at the concrete level. It is appropriate for students to begin drawing solutions to their problems as soon as they demonstrate their mastery of a particular math concept/skill at the concrete level (Witzel, 2005). Multiple opportunities of practice also assist students to internalize the particular problem-solving process. Additionally, students' concrete understanding of the concept/skill is reinforced because of the similarity of their drawings to the manipulatives they used previously at the concrete level.

Finally, at the *abstract stage of understanding* requires students to perform mathematical concepts and skills with numbers and symbols only. Abstract understanding is often referred to as, "doing math in your head" (Witzel, 2005). Completing math problems where math problems are written, and students solve these problems using paper and pencil is a common example of abstract level of problem solving.

When using CRA, teachers make sure that students understand what has been taught at each

stage before moving instruction to the next one (Berkas & Pattison, 2007). How can teacher build up effectively CRA-based lesson with raising students' interest and participation during the lesson?

At first, the sequencing of stage with its relevant activities is crucial as Butler, Miller, Crehan and Babbit (2003) recognized that concrete stage is critical in an effective teaching and learning fraction equivalence by comparing the mean scores between students of CRA and RA groups. Teachers ought to start at the concrete stage before moving to the representational and, finally, to the abstract stages. Secondly, it is also important to ensure that learners acquire, retain and master the specific mathematical skills in each stage of the instruction (Witzel, 2005). It clearly means that understanding in each level of CRA sequence requires active participations from students with their interest and thus teacher must provide that opportunity for students to master and transfer knowledge from one stage to the next. In other words, CRA sequence requires an effective teaching approach for lesson delivery to guide students in the construction of mathematical concept.

2.2 The Structured Problem Solving Japanese lesson pattern

One of effective mathematics teaching approaches for lesson delivery has been identified in TIMSS videotape study. This is the standardized Japanese lesson pattern regarded as structured problem-solving approach (hereafter "Structured Japanese lesson pattern").

Stigler and Hiebert (1997) claimed that aspects of mathematics lessons in Japan as identified had a strong uniqueness in comparison with those of Germany and the United States. One of the striking one in the lessons in Japan lies in its structure and the way of its delivery. The structure of Japanese lessons was characterized as "structured problem solving" where more emphasis is put on fostering the mathematical thinking of students as the main goal for lessons in Japan than in the other two countries (Stigler *et al.*, 1999).

In the structured problem-solving approach, Japanese teachers emphasize that one of the most important roles of the teacher during a lesson is to facilitate mathematical discussion after each student comes up with a solution. When the teacher presents

a problem to students without giving a procedure, it is natural that several different approaches to the solution will come from the students (Yoshida & Sawano, 2002). Thus, the textbooks include examples of students' typical approaches and ideas. As the goal of the structured problem-solving approach is to develop students' understanding of mathematical concepts and skills, a teacher is expected to facilitate mathematical discussion for students to achieve this goal. Towards the end of a lesson, a teacher often leads the lesson to pull all the different approaches and ideas together to see the connection (Yoshida and Sawano, 2002). Then, he or she summarizes the lesson to help students achieve the objective of the lesson. The teacher often asks students to reflect on what they have learned during the lesson (Yoshida and Sawano, 2002). In addition, according to Shimizu (2000), a common organization of lessons is comprised of the following segments such as "presentation of a problem" "problem solving by students" "a whole-class discussion about the methods for solving the problem" and "summing up by the teacher; (exercises/ extensions)". These often serve as the "steps" or "stages" both in teachers' planning and in teaching-learning processes. Thus, this study will practice the CRA sequence of instruction in the development of learning materials while instructional goals will be guided by instructional strategies embedded in the structured Japanese lesson pattern.

3. Research Methodologies

3.1 Selection of school

The study was conducted in March 2017 for three weeks, targeting the students in two 7th grade

classes in one primary school in Port Moresby that the previous survey on PNG mathematics was done in May 2015 (Apule 2017). We selected this school based not only on its academic performance (average level) among schools in Port Moresby, capital city of PNG, but also historical declining in its achievement level since the introduction of OBE curriculum. The geographic as well as socio-economic contexts regarding school location, teacher and students have been already mentioned in the previous paper (Apule 2017).

3.2 Intervention of lessons

Seventy six students (ages 12-15) (38 male and 38 females) and two of 7th grade teachers participated in this survey (male and female). One class was randomly assigned to the experimental class (EC) ($n = 37$), while the other formed the control class (CC) ($n = 39$). The EC students were taught by one of the authors following designed lessons with our new approach (CRA sequence of instruction guided through the structured Japanese lesson pattern). The CC students were taught by one of the classroom teachers using traditional type of regular instructional materials and methods. Each student group participated in three lessons. Each lesson was approximately 40 minutes in length and consisted of topics and objectives for comparison of fraction size, as given in Table 3.1. The details in lesson contents and progress will be indicated later in the section of "Results". As per the research design, the CC students was taught the same fraction lessons in the same order as in the EC students and the same specific problems were used within the lessons for both classes.

Table 3.1 Objectives of three consecutive fraction comparison lessons.

Lesson No:	Sub-Topics	Objectives
1	Comparison of fractions using the unit fractions	By the end of the lesson, students can compare size of fractions with greater than ($>$), less than ($<$) or equal to ($=$) symbols using the idea of unit fractions.
2	Finding Equivalent fractions	By the end of the lesson students can find equivalent fractions by increasing or reducing it with the same factor.
3	Finding Common Denominator to compare fractions	By the end of this lesson, students can compare size of fractions by using the idea of finding a common denominator.

Data source: Apule, 2018

3.3 Data collection and analysis

All the lessons for both groups were audio and videotaped, photographed with observation during the implementation and transcribed. The pretest and posttest required students not only to make answer to each item using the equality or inequality symbols ($>$, $=$ or $<$) to compare the fractions but also explain their answers using words and/or diagrams on the space provided as same as the test used in the previous survey (Apule 2017). Thus, the data collected include the quantitative scoring data of pretest and posttest as well as the qualitative descriptive ones on their strategies or reasoning for their comparison choices. In this way, the study engaged both the quantitative

and qualitative methods of data collection.

All the problems of assessment item are pertaining to comparison between a pair of fractions with different comparison type, as given in Table 3. 2. The level of these problems were 5th grade level in PNG as same as in the previous study (Apule 2017). The structures of assessment items were also same for both pretest and posttest. In the pretest, we used the same numerical values as those used for the previous survey (Apule 2017). In the posttest, the numerical values were slightly changed in all the problem items. Pretest and posttest were conducted for EC and CC students before and after the intervention of three lessons.

Table3.2. Fraction pairs for comparison and their type used in the pretest and posttest.

Item	Pretest	Posttest	Fraction Comparison type
a.	$\frac{5}{6}, \frac{2}{6}$	$\frac{3}{5}, \frac{4}{5}$	Same denominators
b.	$\frac{1}{3}, \frac{1}{2}$	$\frac{1}{4}, \frac{1}{5}$	Unit fractions/ same numerators
c.	$\frac{1}{2}, \frac{2}{4}$	$\frac{2}{6}, \frac{1}{3}$	Equivalent fractions
d.	$\frac{5}{6}, \frac{3}{4}$	$\frac{3}{5}, \frac{2}{4}$	Different Denominators
e.	$\frac{2}{3}, \frac{3}{5}$	$\frac{4}{6}, \frac{3}{4}$	Different Denominators
f.	$\frac{3}{5}, \frac{3}{4}$	$\frac{4}{6}, \frac{4}{5}$	Same Numerators/ different denominators

The quantitative scoring data are used to compare statistically between those of pretest and posttest for EC and CC by the two-tailed test with the level of significance of 0.05 using SPSS version 21.0 (IBM Corp, 2012). The qualitative descriptive data were analyzed on students' reasoning or strategy in comparing fraction size by using an open coding for each response from student. The results were then combined through checking the similarity in the reasoning or strategy they deployed. Subsequently, three main categories were identified as misconception 1 (M1), 2 (M2) and 3 (M3) depending on the characteristics of misconception students made as already mentioned in the previous study (Apule 2017). M1 includes the incorrect students' view that the numerator and denominator are separate whole numbers. It exemplified the answer of one students who described that $\frac{3}{4}$ is less than $\frac{5}{6}$ because '3' is smaller than '5' as well as '4' is smaller than '6'. M2 includes their view that the same numerator fractions are equivalent as exemplified by the answer

of one student who answered " $\frac{1}{2}$ is equal to $\frac{1}{3}$ because the numerators of both fractions are same". M3 includes two types of misconceptions. One is the inappropriate use of multiplication or addition in the comparison of the size of fraction as exemplified by the answer of one students who described " $\frac{5}{6} > \frac{2}{6}$ because ' $\frac{5}{6}$ multiplied by $\frac{2}{6}$ equal to $\frac{10}{6}$ and then $10 > 6$.'" The other is the use of incorrect diagram and/or picture representations.

3.4 Ethical Considerations

We abided by all ethical guidelines and practices as set forth by our affiliated institutions. From this perspective, different approval authorities were involved for the endorsement of the research study. Separate information letters and permission/consent forms were sent to the school head teacher, teachers and parents of children participating in the study. The letters and consent forms gave information relating to the purpose of the study, the types of activities involved, the need for confidentiality and

the management of potential risks. Children were expected to give their written consent by indicating their willingness to participate on a form prior to any research activities. The information on the form explained the testing procedure, as well as their right to withdraw at any stage without any penalty.

4. Results

4.1 Design of lessons for EC

Three lessons for EC was designed based on CRA guided through the structured Japanese mathematics lesson pattern. Although CRA and lesson pattern were integrated into one lesson, for descriptive purpose, we will make separate explanation of how CRA and then

structured Japanese mathematics lesson pattern was reflected concretely on the lesson design.

4.1.1 Reflection of CRA in the lesson design

The 1st lesson consists of the first two out of three stages, concrete and representational stages, as given in Figure 4.1. In this lesson, students learn to compare fraction size using number line and through playing fraction size comparison game. Concrete element of the sequence illustrated by cutting of an apple into fifths in the introductory part to demonstrate the meaning of denominator and numerator of fraction, while the representational elements were set of cards displaying different representations of fractions and unit fractions on the number line.

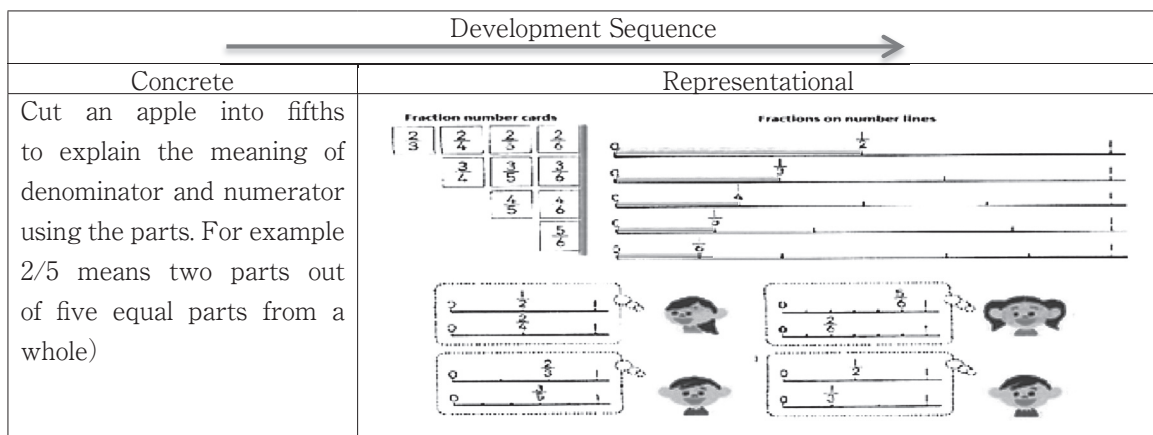


Figure 4.1 Development sequence of the 1st lesson (Data source: Apule 2018)

The 2nd lesson consists of all three stages of CRA sequence of instruction, as given in Figure 4.2. In this lesson, students now are provided with an intermediate step where they transfer their concrete understanding toward an abstract level of understanding mediated by representational learning element. In this lesson, an apple was also used as concrete material to show the equivalent nature of

fraction $1/2$ and $2/4$ concretely before representing them on the number. While the 3rd lesson consists of only “Abstract” stage, as given in Figure 4.3. In this lesson, students now understand math concept and perform mathematical skills at the abstract level. That is, they come to deal with their tasks only using numbers and math’s symbols as called “Doing Math in your head”.

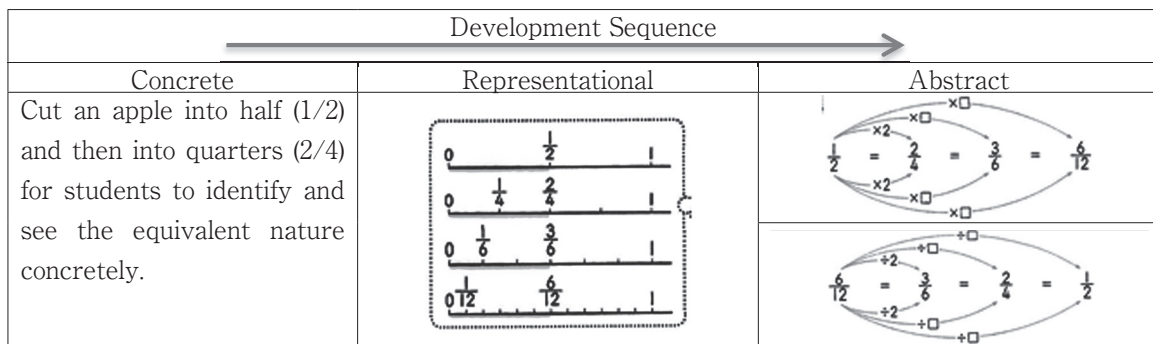


Figure 4.2 Development sequence of the 2nd lesson (Data source: Apule 2018)

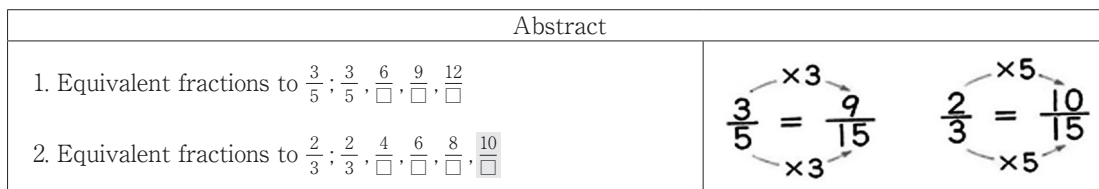


Figure 4.3 Development sequence of the 3rd lesson (Data source: Apule 2018)

The lesson objectives in Table 3.1 were sequentially organized in the design of three lessons for students’ conceptual understanding of fraction and comparison of its size. The CRA sequence of the lessons not only blends conceptual and procedural understanding in a structured way but also each lesson is connected through the overlapped stage(s). Although the learning contents are different from each other depending the objectives, the 1st and 2nd lessons are connected by the first two stages of CRA, and the 2nd and 3rd lessons are connected by the last stage of CRA. These provides a framework for students to make meaningful connections between conceptual and procedural aspects of fraction. The representational (material) element was fractions on number line; displaying unit fractions on a number

line and fraction cards to play fraction comparison game. Other students support materials were also provided such as charts to display fractions and students’ worksheets.

4.1.2 Reflection of Japanese mathematics lesson pattern in the designed lessons

The CRA sequence was guided through structured Japanese lesson pattern to align the flow of the activities in each phase of the lesson. Table 4.1 summarizes the key activities carried out in each of the three experimental lessons. For each lesson, the facilitator was guided by structured lesson and blackboard plan which provides the detailed activities and development of the lesson.

Table 4.1 Japanese lesson pattern and key activities in the lessons for EC.

Teaching and learning activity	Key activities
1: Review of Previous lesson	Students were asked about specific previous knowledge to relate today’s lesson.
2: Presenting the problem for the day	Introduce today’s lesson objective and ask students to think about ways to solve or find the answer.
3: Students working individually or in groups	Students think about ways to solve the problems individually or in groups.
4: Discussing solution methods	Students present/share their ideas on the board for class discussion. Teacher reinforces and facilitates students’ discussion.
5: Highlighting and summarizing the main points	Students asked to recall/reflect on what they have learnt in/during this lesson. Summary notes highlighting the main points.

Data source: Apule, 2018

4.2 Lesson intervention

4.2.1 Lesson deliveries for EC

The lessons for EC were designed and delivered as already given in Table 4.1 by one of authors, according to the lessons designed based on CRA sequence of instruction, as given in Figure 4.1, Figure 4.2 and Figure 4.3. The outline of intervention of three lessons for EC was as follows;

Checking students’ previous knowledge in the introductory part was very specific to the lesson to be taught. Then in the development part, students

were asked to work in small groups on practice problems designed to consolidate the lesson content. When the students worked on the practice problems, the teacher interacted with the students by providing guidance and responding to their questions. Whole-class discussions then followed where students would share their solutions with other members of the class. In all contexts, the teacher’s explanations, questions, and responses highlighted connections between concepts and symbols whenever possible. The lesson summary was made from the students’ reflection

on what they have learnt through the lesson as an evaluation for the teacher to brainstorm the main idea and check whether the lesson objective was achieved or not. In the 1st lesson, through pairs of students, students engaged in a game using fractions cards and fractions on the number line sheet to compare two fractions (*Concrete to Representational*). Using students' knowledge learned in the 1st lesson, in the 2nd lesson, they thought about how to make equivalent fractions using unit fractions on the number line to discover methods to various equivalent fractions with different combination of denominator to denominator (*Representational to Abstract*). The types of practice problems given to the students in EC required them to practice making connections between representational model and mathematical symbols. For example, on a worksheet on equivalent fractions, the students were required to write the 'rule' or 'method' for finding equivalent fractions and then produce equivalent representations for a number of fractions. Thus provided them to make connections with the representational model (number line) and abstract nature which gave the conceptual meaning to the procedure they were using. In the 3rd lesson, the teacher proceeded to the abstract level, that is, for the students to compare fractions which are different in both the values of denominator and numerator such as $\frac{3}{5}$ and $\frac{2}{3}$ using the idea of equivalent fraction that they learnt in the previous lesson. In this instance, only numbers (*abstract only*), mathematics rules and symbols were used. Learners were afforded many opportunities to practice and demonstrate their mastery of the concepts being taught.

Based on the reflection on lesson intervention by one of authors who conducted three lessons, the specific features of these lessons for EC are as follows:

- The objectives of lessons were closely linked through CRA sequence-based lesson design.
- The connection between the models (card, number line) used and fraction was clearly explained.
- Primary teaching activity was facilitating student to think as well as to discuss the problems and its solutions provided by teacher.

4.2.2 Lesson delivery for CC

Regular type of lesson delivery was observed in all the three lessons for CC. Based on the observation

by one of authors, the outline of lessons for CC was as follows:

Each lesson started with a short introduction during which the teacher identified the objective of the lesson and instructed students to retrieve required materials (e.g., notebooks, pencils, or other tools). Each lesson began with teacher asking very broad introductory questions and writing today's lesson topic on the board. This was then followed by two or three problems that were solved by the teacher in front of the entire class to show the procedures how to get the correct answers of those problems. Students are then given a series of practice problems to solve following the procedure set by the teacher.

More than half of lesson time for each class period was observed students working on practice problems. They were not provided enough time to think and accordingly have to follow the correct procedure to get the right answer for the practice problems. In the conclusion of the lessons, teacher provided all the answers of the practice problems and asked students to check against the answers provided with theirs and make correction accordingly. Lessons were closed with collecting students' notebooks by teacher as observed in two of the three lessons. Based on the observation by one of authors, the specific features of lessons for CC are as follows:

- Teacher did not indicate any relationship between the three lesson objectives.
- Teacher used the rectangular region model but its connection or meaning between the model and fraction was not explained.
- Primary teaching activities were largely focused on procedural aspect of fraction topics and students are just following the procedures delivered by teacher.
- Teacher did not show any connection between the procedure and its conceptual background.

4.3 Quantitative analysis

The pretest and posttest were conducted before and after the intervention of three lessons for EC and CC. The answer was judged as correct only if both the choice and its reasoning are correct as the examples, indicated in Figure 4.4. Then the correct answer rate was compared pretest and posttest result between EC and CC.

As given in Table 4.2, in both EC and CC students,

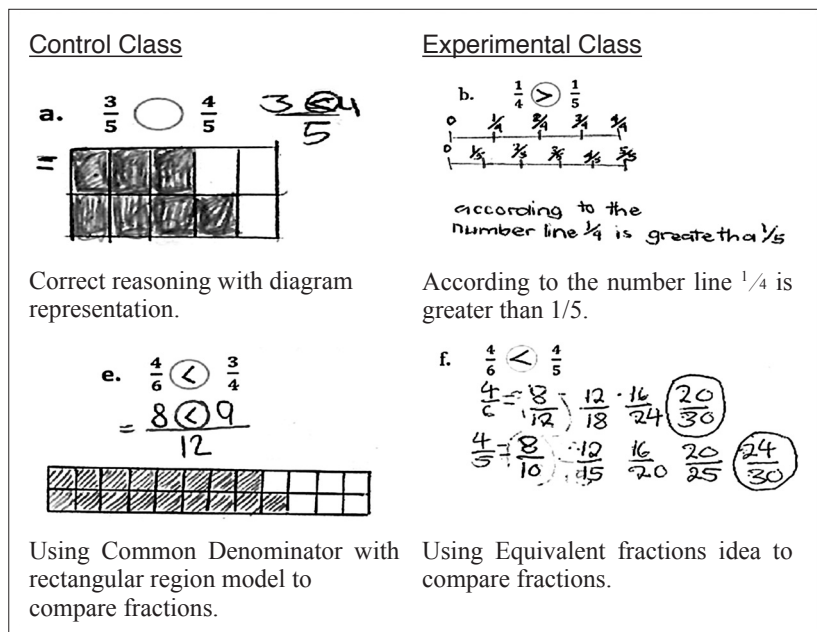


Figure 4.4 Examples of correct answer from posttest (Data source: Apule, 2018)

the highest correct answer rates of 49% (CC) and 38% (EC) were resulted in item 'a'- comparison of fraction with the same denominator- in the pretest. The other five comparison items (various pair of fractions with different denominator) were poorly performed. The overall performance was very poor for both EC and CC students in the pretest with the same average score 0.65 point for EC and 0.97 point for CC). The difference is not statistically significant ($\rho = 0.170$). It indicates that both EC and CC students were nearly the same in their understanding of comparison of fraction size before our intervention. In the posttest, both the CC and EC show great improvement after

receiving the three respective lessons. Additionally, EC indicates higher correct answer rates in all the items than those of CC. Except for item 'a' ($\rho=0.286$), the differences in correct answer rates in the five items ('b'~'f') are statistically significant between EC and CC (for items 'b', 'c', 'd', 'e' and 'f', ρ value is 0.012, 0.002, 0.007, 0.04, 0.006, respectively). The difference in average score of posttest between EC (4.59 points) and CC (3.03 points) is also clearly seen with statistical significance ($\rho= 0.001$). In addition, comparison of CC and EC about the difference between the pretest and posttest shows a significant difference at $\rho = 0.000$, too.

Table 4.2. Comparison of correct answer rate (%) and average of pretest and posttest for EC and CC.

Class	Test	Question item						Average test score
		a	b	c	d	e	f	
EC	Pretest (%)	38	11	0	8	5	3	0.65
	Posttest (%)	86	81	81	81	62	68	4.59
CC	Pretest (%)	49	15	10	18	3	3	0.97
	Posttest (%)	77	54	49	51	38	36	3.03

4.4 Qualitative analysis

As per the open-ended test design, most students from both EC and CC attempted to write down their reasons or strategies for their choices. To see the impact of our intervention, we calculated the rate of students who appeared to have misconception categorized as M1, M2 or M3 to the total number of students in each class by category for each item of

pre- and posttest, as given in Table 4.3.

It can be seen from Table 4.3 that students from both groups put in tremendous effort to dismiss M1 observed in the pretest. The M2 type of misconception was observed only in items 'b' and 'f' in both the pre- and posttest because of the nature of the fraction pair in these items. Though both groups showed the reduction in this misconception view in the posttest,

the EC students show significant decrease in its rate in item 'b' from 57% to only 8% as well as in item 'f' from 54% to only 5%. But a considerable number of students of CC upheld still M2 type misconception in item 'b' from 33% to 18% and in item 'f' from 36% to 26%. It indicates that students of EC showed much greater decrease in the rate of M2 type of misconception after lesson intervention in comparison with those of CC.

In the posttest, the EC students showed significant improvement for M3 type of misconception on questions items 'a', 'c' and 'd' based on the decrease in the rate from 38% to 14%, from 38% to 16% and 32% to 19%, respectively. But CC students do not show any improvement based on the changes in the rates in terms of these items. They represent either unchanged (items 'a' and 'c') or the increase (item 'd') in the rate. Contrarily, not decrease but the slight

increases in the rates for both the question items of 'e' and 'f', comparison of fractions with different denominator, are seen in the students of both EC and CC. In item "b", not any change and a slight increase in the rate is observed in EC and CC students, respectively. Based on their descriptive answers, their errors could be accountable for a simple miscalculation of multiplication in the reduction process of common denominator before comparison that has been taught in lessons 2 and 3 in EC students. Typical M3 type of misconception in CC students was due to an incorrect representation (rectangular region model) as well as an inappropriate approach in comparing fraction using addition or multiplication. They also committed procedural errors in the reduction of fractions to a common denominator due to the miscalculation in multiplication as same as EC students did.

Table 4.3: Rate of students who have misconception by category in each item of pre- and posttest.

Category of misconception	Item	Experimental class (n=37)		Control class (n=39)	
		Pretest	Posttest	Pretest	Posttest
		Rate in %	Rate in %	Rate in %	Rate in %
M1	a	24(9)*	0(0)	28(11)	0(0)
	b	22(8)	0(0)	28(11)	0(0)
	c	62(23)	3(1)	44(17)	5(2)
	d	62(23)	0(0)	49(19)	10(4)
	e	62(23)	0(0)	54(21)	10(4)
	f	32(12)	8(3)	33(13)	5(2)
M2	a	0(0)	0(0)	0(0)	0(0)
	b	57(21)	8(3)	33(13)	18(7)
	c	0(0)	0(0)	0(0)	0(0)
	d	0(0)	0(0)	0(0)	0(0)
	e	0(0)	0(0)	0(0)	0(0)
	f	54(20)	5(2)	36(14)	26(10)
M3	a	38(14)	14(5)	23(9)	23(9)
	b	11(4)	11(4)	23(9)	28(11)
	c	38(14)	16(6)	46(18)	46(18)
	d	32(12)	19(7)	33(13)	38(15)
	e	32(12)	38(14)	44(17)	51(20)
	f	11(4)	22(8)	28(11)	33(13)

* Numbers in brackets indicate the actual number of students who have misconception

5. Discussion

5.1 The impact of our new approach to teaching and learning fraction in the context of PNG

Irrespective of teaching approaches used in this study, quantitative data analysis on both pretest and

posttest scores clearly and strongly suggested that the lesson interventions not only in EC but also CC improve students' understanding of how to compare fractions in terms of their size. Nevertheless, EC students showed much more improvement than CC students on average as well as in almost all item with a

statistical significance except for item 'a'. These could point out two issues in terms of teaching and learning of fraction at the primary school level in PNG. Firstly, to a lesser or greater extent PNG students have the potential to respond to a lesson if a teacher teaches them carefully even through a regular approach. Secondly, in general, our new approach, CRA sequence of instruction guided through structured Japanese lesson pattern, would be more effective to improve students' conceptual understanding in terms of comparison of fraction size than the regular approach since the test scoring data were backed up not only by just students' choice of which fraction is bigger than the other in a certain fraction pair but also by their descriptive or illustrative reasoning why they made the decision of their choice.

5.2 The impact of our new approach depending on the type of students' misconception

In the previous study, one of authors recognized the three types of misconception in the descriptive responses from students to the same pretest problems as used in this study (Apule 2017). Comparing the extent of change in the rate for each misconception type between pretest and posttest in each EC and CC students, it was suggested that the impact of our new approach was different in its extent depending on the misconception type (Table 4.3). The M1 type misconception has already been concerned as one of serious problems in teaching and learning fraction for long time (e.g. Lukhele, Murray & Olivier 1999; Murray, Olivier & De Beer 1999) since it is the most fundamental concept that a denominator and numerator is not a separate whole number, but both are part of fraction as a rational number. To the correction of M1 type misconception, although even in CC students, an appreciable improvement could be seen, it was revealed that our new approach dismissed this type of misconception almost perfectly. While, the appreciable specific impact of our new approach was resulted in the correction of M2 type misconception: the fractions with same numerator is equal in their size.

Differently from the misconceptions of M1 and M2 types, to that of M3, the impact could be seen to some extent, but it was limited only to the half of six test items. Although it may be necessary to examine more details in students' incorrect reasoning that

have been categorized as M3, one could find possible solution to rather irregular result in the difference in the characteristics between M3 and both the M1 and M2 types of misconceptions. Both the misconceptions of M1 and M2 are closely related to the conceptual understanding regarding of the relationship between a numerator and a denominator of fraction as a rational number. But M3 type of misconception could be conceived as a procedural rather than a conceptual in its characteristic, as described in the "Result". As the procedural misconception is not always related to only the conceptions of fraction but to another mathematical concept and/or skills, for example those related to calculation of whole number, we need to consider how to raise them in teaching and learning of mathematics at lower grade levels.

5.3 Implication of our study in the context of PNG

Based on the outcomes from our study on new approach to the concept-based teaching and learning mathematics at primary school level in PNG, it is largely expected that CRA sequence of instruction could provide the useful framework for lesson design as well as for its delivery to create a meaningful mathematics lesson for students. It could largely encourage students to acquire the abstract level of mathematical knowledge based on their reality in their daily life. Thus, the PNG primary school level mathematics should deploy this framework to guide every aspect of current educational reform from the administrative levels of the alignment of mathematics curriculum, development of textbook and in-service and pre-service training to the implementation level of teaching and learning in a classroom. In particular, it is worthwhile that professional development in both in-service and pre-service training for mathematics teaching should focus not only on just understanding of the content and teaching material development but also on the instructional methodologies in close relation to content and material within the framework of CRA. By contextualizing the situation in a classroom in PNG, our study has conclusively proved that the Japanese type of teaching methodology as "the structured problem solving-approach of Japanese mathematics lesson pattern" was workable within the framework of CRA for teaching and learning mathematics. It would be one of the most promising instructional methodologies for PNG mathematics. Based on these

identified components and through contextualization, PNG needs to adopt CRA sequence of instruction as a standard framework in combination of structured Japanese lesson pattern as instructional methodology and use at the primary school level in the standard based mathematics curriculum. In addition, The PNG QUISME project would be expected to capture these important components through the development of mathematics teacher's guide through technical assistance from JICA experts. For such a development, for example, the work of Witzel, Riccomin and Schnider (2008) in terms of the curricular CRA modification in secondary mathematics may be supportive. But in any way, as the actual implementation is a major challenge ahead, it is strongly recommended that the training for teachers as well as teacher candidates must be provided effectively and in the transition phase from the OBC to SBC.

6. Limitation

As the recent research (Apule, Ishizaka, Ozawa & Kozai 2016) showed that the urban school performance level was just the same as those of rural and semi-urban schools, this study could signify the level of mathematics education in primary school in PNG. But it focused only on fraction comparison topic. A further study could include more topics such as measurement, geometry, and statistics and should involve more schools/institutions not only in urban but also semi-urban and rural area.

7. Conclusion

Given the demand by the current PNG national education curriculum that requires much higher quality in teaching and learning of important subject such as mathematics, one of the biggest challenges is to provide effective mathematics instruction for students at the primary school level. Selecting and implementing instructions that incorporate applicable materials and methods for mathematical concept-based teaching is one way to achieve the intention of national education curriculum. This study indicates the instructional intervention such as CRA sequence guided through the structured problem solving Japanese mathematics lesson pattern is one of promising way to promote students' thinking and

enhance mathematical conceptual understanding. Hence, in the light of the current educational reform to the SBC in PNG, this study strongly proposes a drastic change in the current culture of mathematics instruction to concept-based teaching through applicable materials and methods, which is fundamental to the overall success of the reform in PNG's national education system.

Acknowledgement

We would like to express our deep gratitude to the support and funding of the Japan International Cooperation Agency (JICA) and PNG Department of Education (NDoE) for the support of this study. Finally but not the least, we also grateful to all the participating children and teachers of the school where this research was carried out. You made this research possible.

This paper has been drafted through revising the master thesis of Apule submitted to the Graduate School of Naruto University of Education.

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