

# **A Study to Foster Proactive Learning in Mathematics at Primary Schools in the Republic of the Marshall Islands through Concrete-Pictorial-Abstract (CPA) Approach**

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## **Abstract**

Marshall Islands Standard Achievement Test (MISAT) (2018) clarifies that geometry is always the weakest mathematical area among most of the public primary and secondary schools in the Republic of the Marshall Islands. The objective of this article is to foster Proactive Learning in Mathematics at Primary Schools in the Republic of the Marshall Islands through Concrete Pictorial Abstract (CPA). This article also indicates the situations of what worked and what did not work in the study. A total of eighty nine students from two primary schools participated in this study. However, due to the issue of absenteeism, the actual sample population was eighty students retrieved from thirty six students of school A (CC = 18 and EC = 18) and forty-four (CC = 24 and EC = 20) from School B. The experimental class (EC) and the controlled class (CC) were both taught by the author. The lesson structure of the EC composed of concrete-pictorial-abstract guided through collaboration and Japanese structured lesson format. However, the lesson structure of CC entailed lecturing and collaborative work. It appeared that there is no big difference between the average score of the EC and the CC in the pre-test and the post-test. The importance of CPA is to enhance proactive learning where students can proactively manipulate materials and have visualization while going toward abstract. Within each stage, it is dominant that every student understands basis and meaning of it. Using Japanese teaching approach through the manipulation of materials as scaffolding resulted as an effective approach to enhance students' higher level thinking skills.

## **1. Introduction**

### **1.1 Background**

Geometry had always been the lowest area at the duty station and most public schools in the Marshall Islands (MISAT, 2018). Despite teacher workshops and student-teacher interaction, students continue to struggle in this field during standardized assessments. One of the reasons that contribute to this issue is that teachers are not well prepared. In addition, the traditional teaching approach which is "chalk, talk, and giving answers" contributes to this crisis. The author, as a primary school mathematics teacher, empirically found out that students are very weak in

using higher level thinking skills because teachers usually give the answers and focus more in the abstract and the product. The author also discovered that teachers rarely use materials to teach mathematics lessons. Marshall Islands Standardized Achievement Test (MISAT, 2018) indicates that students have low conceptual understanding regarding geometry. As of 2018, Figure 1 shows sixth graders' result of all the schools at the Marshall Islands on a summative assessment. Below in Figure 1.2, the benchmarks are indicated. For example, M.6.2.1 means Math – Grade 6- Standard 2, and Benchmark1. Standard 2 is mostly about geometry.

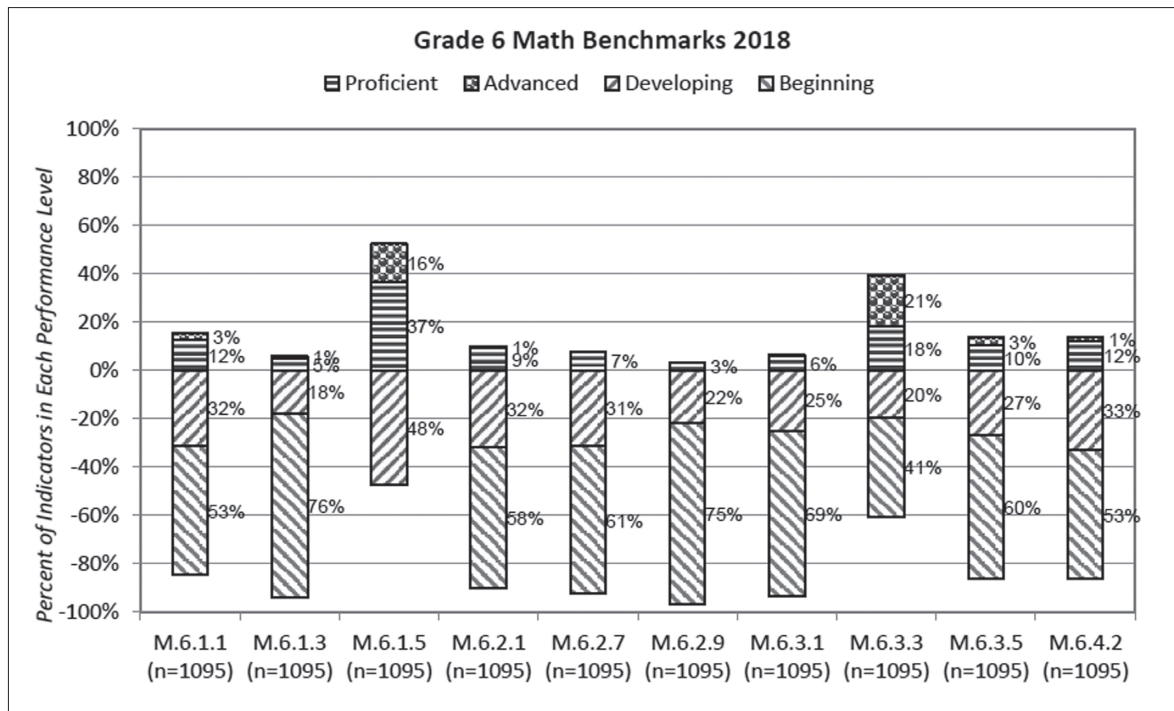


Figure 1: Grade 6 Math Performance Level 2018 (MISAT, 2018, p. 18)

M.6.2.1	<i>Identify the relations among angles</i>
M.6.2.7	<i>Find the areas of plane figures and circles</i>
M.6.2.9	<i>Find surface area and volumes of solid figures</i>

Figure 1.2: Grade 6 Math Benchmarks (summarized by the author)

### 1.2 Why Singaporean Approach (CPA) and Japanese Style Approach?

At the public primary schools in the Marshall Islands, students do not have ample of opportunities to manipulate materials to understand mathematical concept and application. Furthermore, students are always spoon-fed by the teaching approach of primary teachers. According to Stigler and Hiebert (1997), Japanese mathematics lessons as “structured problem solving” is very effective to foster higher thinking skills. Concrete-Pictorial-and Abstract (CPA) approach contains three stages known as ‘doing, seeing, and abstract.’ Jordan et al. (1998) declared that CPA has major influence on students with disabilities and with low performance. It would be hypothesized that by integrating CPA approach and Japanese Teaching Approach, primary students in the Marshall Islands would have more opportunities to practice higher thinking skills with the support of using the three stages.

## 2. Theoretical background

### 2.1 Concrete-Pictorial Abstract (CPA)

Concrete-Pictorial-and Abstract (CPA) is a Singaporean teaching approach for mathematics. CPA contains three indispensable instructional steps which link each other, known as Concrete, Pictorial, and Abstract. According to Hong, et al. (2015), CPA is a gradual systematic approach where each stage builds on to the previous stage and therefore must be taught in sequence. The first stage is known as the “doing” stage and it involves physical manipulation for solving a math problem. The pictorial or semi-concrete stage is the second stage. It is also known as the “seeing” stage where images are used to represent mathematics problem to solve. Lastly, abstract or symbolic stage involves only numbers and symbols to solve the math problems.

Concrete-Representation-Abstract, Concrete-Semi-Concrete-Abstract, and Concrete-Representation-Verbal-Abstract are other names for CPA which imply the same meaning. An article by Jordan, Miller, & Mercer, 1998; Sousa, 2008 also proves that the CRA [CPA] sequence has been shown to be particularly effective with students who have difficulties with mathematics. The CRA approach has also been employed to aid students with learning disabilities to learn mathematics; CRA has been reported to be

effective in remediating deficits in basic mathematics computation (Morin & Miller, 1998), in fractions (Butler, Miller, Crehan, Babbit, & Pierce, 2003) and algebra (Witzel, Mercer, & Miller, 2003). Concrete-Pictorial-and Abstract (CPA) is a systematic teaching approach which is efficient for primary, lower secondary, and even secondary.

## 2.2 Collaborative Learning

As stated by Graesser et al. (2018), collaborative skill is an essential skill at home and the community. In fact, much of the problem solving and decision making in the modern world is performed by teams as stated by Graesser et al. (2018) for National Research Council (2011). It is also mentioned that the uncooperative members can be a threat to the success of a team, but with the good leadership of effective leaders, they can change the mindset of uncooperative colleagues. Collaboration and social communication facilitate productivity in [most] workplace as Graesser et al. (2018) clarified for (Klein et al. (2006); Salas et al. (2008), engineering and software development (Sonntag and Lange 2002). Therefore, discussion about including more group-based project-based learning as well as teaching and assessment of collaboration as part of 21st century is considerably facilitated by national education systems.

## 2.3 Japanese Teaching Style

Japanese mathematical lessons entail more of problem solving, especially in the primary level (Takahashi, 2006). Stigler and Hiebert (1997) described Japanese mathematics lessons as “structured problem solving” which is very effective to foster higher thinking skills. Takahashi (2006) declared that Japanese instructional approach, structured problem solving, is designed to create interest in mathematics and stimulate creative mathematical activity through students’ collaborative work. In this approach, the lesson usually starts with students working individually to solve a problem. After that, their approaches and solution are shared to the whole class for discussion. Teachers appear to take fewer roles allowing their students to invent their own procedures for problem solving. On the other hand, teachers carefully design and orchestrate lesson so that students are likely to use the procedures that have been developed recently in class (Stigler & Hiebert, 1997).

## 3. Research Purpose and Methodology

### 3.1 Research Purpose

The objective of this article is to foster Proactive Learning in Mathematics at Primary Schools in the Republic of the Marshall Islands through Concrete Pictorial Abstract (CPA). For example, using Singaporean teaching method called Concrete-Pictorial-Abstract (CPA) and integrating Japanese Teaching Approach is the aim of this study to upgrade students’ geometrical skills. To be more specific, the lesson implementation had to include manipulates and the structure of the lesson will be followed by Japanese teaching approach where students will discover for themselves while the teacher will be a facilitator. Specifically, finding proactive and effective teaching methodologies that can gradually increase students’ mathematical skills in these areas is the main focus of this study. These are the questions to consider under the main goal;

1. *Does applying Singaporean Approach - Concrete - Pictorial - and Abstract (CPA) based on Bruner’s philosophy through constructivism effective?*
2. *Can Japanese Teaching Approach enhance higher level thinking skill of Primary students in the Marshall Islands?*
3. *Is implementing geometric lesson through collaborative learning effective?*

### 3.2 Research Methodology

In this study, two primary schools were randomly selected to participate. Both schools were in the same atoll, Majuro, which is the Capital City of the RMI. These two schools were selected randomly because the second part of the study will be conducted in School A where most of the students from these schools will attend as a transition from six graders to seven graders. School A is composed of only seven graders and eight graders, and most of the students come from five public primary schools out of nine public primary schools in Majuro. In each school, there was an experimental class (EC) and a controlled class (CC), and the lessons were conducted to sixth graders. Before and after the implementation of the lessons, pre-test and post-test were given to detect the effect of the study and the misconceptions that students had been struggled with.

### 3.2.1 Intervention of lesson

A total of eighty nine students from both schools participated in this study. However, the issue of absenteeism leads to the actual sample population of eighty students retrieving from thirty six students of school A (CC = 18 and EC = 18) and forty-four (CC = 24 and EC = 20) from School B. The experimental class (EC) and the controlled class (CC) were both taught by the author. The lesson structure of the experimental class was composed of concrete-pictorial-abstract guided through collaboration and Japanese structured lesson format. However, the lesson structure of control class (CC) entailed lecturing and collaborative work. There were three lessons implemented in the topic of; area of triangle, net of prism, and surface area. These topics were selected because primary students in the Marshall Islands have major issues in them as clarified by the MISAT Results (2018). In addition, these topics are for six graders in which students have always struggled with. These topics also contribute to real world situations which can help students to recognize the importance of mathematics.

#### 3.2.1.1 Experimental Lesson Structure

In the first lesson of EC, students worked with the group to find the area of a rectangle as an assessment of prior knowledge. Each group had different rectangle. Within their group, they were given a grid rectangle that is equivalent to the rectangle in the prior assessment to cut diagonally to form two equivalent triangles. This is where the ideology of concrete was introduced in this lesson. By themselves, students investigate the relationship



Figure 3.2.1: Evidence of Concrete Stage Activity

between the rectangle and the one that was cut into two congruent triangles. **Figure 3.2.1** below shows what students did in the manipulation stage.

Using a table as a pictorial to model their understanding, students wrote the relationship between the rectangle and the triangles formed from it. The figure 3.2.2 below was used in the lesson as a representation for understanding the concept and to build understanding of the relationship between the

Group Name	Date	Grade
NO.	1	
Area of Rectangle		
Rectangle and Triangle's relation		
Area of Triangle		
Area of Triangle's Formula		

Figure 3.2.2: Sample evidence of pictorial stage

rectangle and the triangles formed.

In this lesson, there were four groups which had different rectangles to cut and to relate to the original rectangle. After students found the area of the rectangle, relation between rectangle and triangles, and area of triangle, they presented their result for discussion. Lastly, by themselves, students detected the reason behind the formula of the area of triangle which is half of the area of a rectangle (area =  $\frac{1}{2}$  x base x height). Students then used the formula to abstractly calculate the area of right triangles. To conclude the lesson, students make a summary stating that the area of a triangle is half of the area of a rectangle.

The second lesson was composed of concrete and pictorial stage as illustrated in **figure 3.2.3** below. Students used magnetic triangles, squares, and rectangle to form rectangular and triangular prism and cube. Students opened the prism to see the net, and then roughly draw the net. With the manipulation of the materials, the author perceived that students were more engaged as compared to the control class. Through visualization and manipulation of materials, students built solid conceptual understanding of net as they were collectively interacted within each group. The third lesson, surface area, was composed of pictorial and abstract since it was a continuation lesson of the previous lesson. In the activity, students roughly draw the net and find the area of each faces.

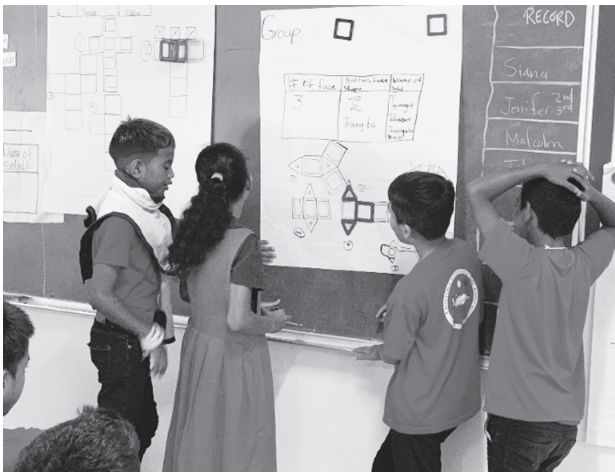


Figure 3.2.3: Sample evidence of Concrete & Pictorial Stage

Within each lesson, students had the opportunity to present their result for the whole class.

### 3.2.1.2 Japanese Lesson Structure in EC each Lesson

The major focus in following Japanese lesson structure was to become a facilitator and to let students to think for themselves. The aim for following this approach is to enhance higher level thinking skill of primary students by giving guidance through manipulation of materials. As a result, the author realized that despite this handy guidance, most students still had difficulties which led him to guide through questioning as scaffolding. Therefore, by themselves through scaffolding, students were able to detect the reason behind the formula for the area of a triangle. Moreover, students were also able to identify several nets of rectangular and triangular prism and cube. The author noticed that this approach was quite challenging because students rarely had chances to be taught in this way. Although it was a challenge, more students in the EC grasped the content and mathematical application for a longer period compared to the students in the CC. The Japanese Teaching Approach is a new teaching model for primary students in the RMI which is quite demanding, but with continuous and consistent practice can get them acquainted with this teaching approach.

### 3.2.1.3 Lesson Structure of Controlled Class

The structure of the first lesson for the controlled class (CC) composed of prior knowledge activity, input, group work, independent, and assessment. After prior knowledge activity, the teacher abstractly did some examples on the computation of the area of a triangle. Then students looked at the examples from the board to work with their group for group presentation. Instead of visualizing the reason behind the formula of area of a triangle for themselves through the manipulation of materials, students were told. After group presentation, students worked individually and were assessed. The benefit of the teaching approach for the control class was that it was much easier to implement. Since most of the parts were done by the author, the learning progress was smooth and fast. Even if the teaching approach was easier, it was meaningless for students because they lack the opportunity to exercise their higher thinking skills. Moreover, the author found out that spoon-feeding leads to short term memory.

## 4. Data Collection and analysis

A pre-test and post-test was given to students to detect the effectiveness of the lessons and to reveal any misconception. A sample population of eighty students (CC = 42 and EC =38) was able to take the pre-test and the post-test. In **table 4.1** below, it was found that i) in both CC and EC, the pre-test score was very low and not so much difference of pre-test score, ii) big improvement between pre-test score and post-test score, iii) in both groups, there was big improvement from pre-test score to that of post-test score, iv) the post-test score's standard deviation of both groups was big enough compared with average test score, which implies that there would be a huge gap among students in both groups.

Using t-test: Two sample Assuming Unequal Variances in MS excel, the author compared two groups' means difference of pre-test and post-test which is illustrated in the Figure 4.2. Statistically, the

Table 4.1: Average & Standard deviation of Pre-test & Post-Test of CC and EC

	NO. Students	Pre-Test Average	Pre-Test SD	Post-Test average	Post-Test SD
Control Class (CC)	42	0.97619	1.023816	7.357143	5.178857
Experimental Class (EC)	38	0.763158	0.91339	6.131579	5.199642

p-value was (0.45) which is greater than 0.05, therefore there was no significant difference of improvement in pre-post tests between CC and EC. The result also indicates that the average difference of the pre-test and the post-test of control class seem to be a little bit higher than the experimental lesson.

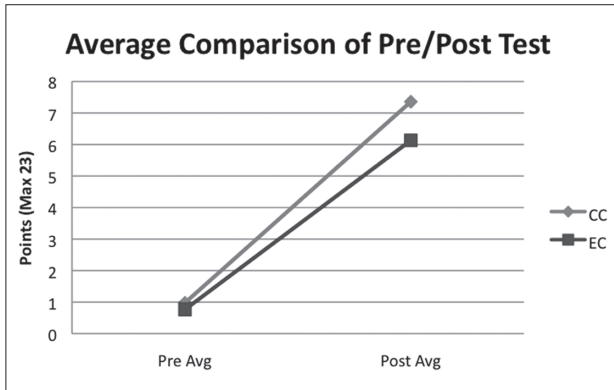


Figure 4.2: Average Comparison of Pre-Test and Post-Test Score

Although the average improvement of the CC and EC was not significantly different, result on reasoning skills of CC was lower than that of EC. The question; “why is the formula to find the area of triangle is  $\frac{1}{2} \times \text{base} \times \text{height}$ ?” was introduced as one of the items of the pre-test and the post-test. The figure 4.3 below shows the result of this question for students who were able to detect the reason behind the formula of the area of a triangle.

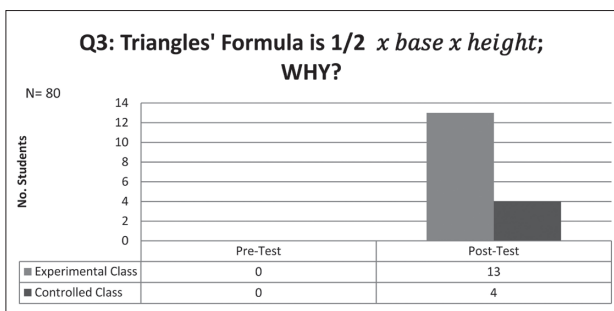


Figure 4.3: Result on the Question: Why Triangles' Formula is  $\frac{1}{2} \times \text{base} \times \text{height}$ ?

As indicated in the chart above, no students were able to detect the reason behind the formula of the area of a triangle in the pre-test. However, in the post-test, both of them increased, but the experimental class shows much more increment as compared to the control class. It implies that through CPA and Japanese teaching approach, most students in EC

were more engaged and they spent time to think individually and collaboratively to understand the reason why the formula is formed in that way. Although it was difficult for them since students rarely had the opportunity to think for themselves, the manipulation of materials was an effective guidance.

Through questioning, it was found that the integration of CPA and Japanese teaching approach somehow helped students to acquire the mathematical application and reasoning longer than the students in the control class. This was proven by the question number three which was introduced in the first lesson. Students in the control class were told about the reason behind the formula of triangles, but students in EC discovered the reason by themselves through manipulation of materials. As indicated in figure 4.3, most students in the EC still held on to the reason behind this formula compared to the students in the CC. Figure 4.3.1 indicates the work of students of CC in the pre-test and post-test. The picture on the left (L) indicates the work in the pre-test and on the right (R) is the work from post-test. As indicated in the figure 4.3.1 at left photo, student had no idea about the formula and he/she thought that to get the area, the base and the height must be multiplied only. In the post test, this student did well in the computation part, but could not respond correctly to the question about why the formula is  $\frac{1}{2} \times \text{base} \times \text{height}$ . Instead, the student wrote, “Because [it] is the formula of a

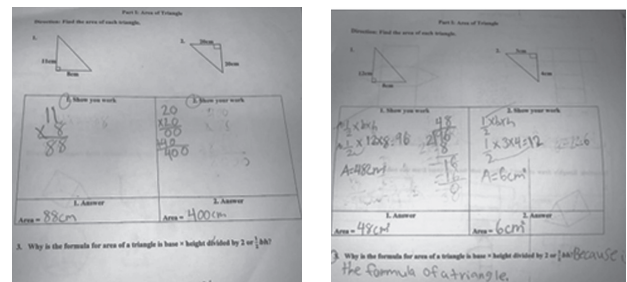


Figure 4.3.1: Photo of student A's work in CC at Pre-test (L) and Post-Test (R)

triangle.

Figure 4.3.2 indicates the work of student of EC in the pre-test (L) and the post-test (R). In the pre-test, this student thought that to find the area of the triangles, he/she just need to multiply the base and the height. The student answered the reason for the formula of a triangle as follows, “Because it has to be

in order.” In the post-test, positive effort was shown, and the answer to question 3 is “because havle [half] of a square or rectangle is a triangle and if you cut the redangle [rectangle], its [it’s] a triangle and put it back

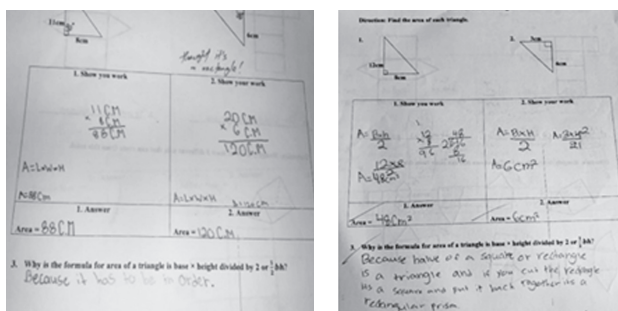


Figure 4.3.2: Photo of Student A’s work in EC at pre-test (L) and Post-test (R)

together, its [it’s] a redangulare [rectangle].

### 5. Limitation

One of the reasons contributed to lower average of experimental class than control class was unbalanced academic performance of students. The selected schools and the sections were randomly selected in which the author found out that most of the students in the experimental class have lower academic performance than those in the control class. Despite this, there is no big difference between the average performance of students in CC and EC in the post-test. In addition, due to the limitation of time, the EC had to rush to complete the lessons; therefore there was less time to spend in the abstract stage.

These lessons should be two-day type of lessons; however, due to limitation of time, they were shrunk to be a one-day type of lesson. Consequently, the time spent in each stage of the CPA and integration of Japanese Teaching approach was not quality. The manipulation of materials and giving time for students to think individually and respond collectively entail a quality of time. The author witnessed that the Japanese Teaching approach was effective. However, because students are not acquainted with, it took more time despite the ideology of using materials and questioning as scaffolding.

### 6. Conclusion

The integration of Concrete-Pictorial-and Abstract and Japanese Teaching approach somehow showed a

great impact in mathematical content and application. Spending quality of time within the three stages of the CPA is vital. The importance of CPA is to enhance proactive learning where students can proactively manipulates materials and have visualization while going toward abstract. Within each stage, it is dominant that every student understands basis and meaning of it. Using Japanese teaching approach through the manipulation of materials as scaffolding is an effective approach to enhance students’ higher level thinking skills. In fact, students had difficulties with this approach because they are not familiar with. Despite a short period of introducing this approach, one of the items in pre-test and post-test showed a huge influence for students in the EC. Implementing lessons through the Japanese teaching style consistently would help students to get familiar with and be able to utilize higher level thinking skills. It is vital to give opportunity for students to think for themselves to upgrade their reasoning and application skills. However, letting students to struggle is not an effective approach. Therefore, it is crucial to guide by asking questions or giving hints *instead of giving the answer*. Though, it was obvious that the reasoning and thinking skills of the EC students showed a considerate impact as shown in figure 4.3, 4.3.1, and 4.3.2 in the Data collection and Analysis section above. The integration of Concrete-Pictorial-Abstract (CPA) and the Japanese Teaching Approach showed a great impact in higher level thinking skills. Therefore, the effort to improve proactive learning in mathematics teaching and learning is still ongoing.

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