

Analysis on Lower Graders' Mathematics Textbooks in Senegal, Japan and Singapore, in Application of Spiral Structure of Its Contents and Concrete, Pictorial and Abstract (CPA) Approach

Abdoulaye FAYE

Naruto University of Education

Abstract

This article analyzes how contents are selected and articulated in and across lower graders' mathematics textbook in Senegal, Japan and Singapore, with consideration to Spiral Progression Approach (SPA) and Concrete, Pictorial and Abstract (CPA) approach. Generally, textbook users pay more attention to the knowledge and skills to teach than the logical sequence of contents. Among other factors, the quality of a mathematic textbook, especially for beginners, is largely determined by the organization and presentation of contents with regard to learners' developmental age.

The overarching goal of this research is to highlight the coherence of content organization and to propose some suggestions in order to improve the quality of textbooks.

In this study, a sample of approved textbooks was taken in these three countries. After a summary of the big ideas fostered by these two approaches, the textbooks were compared to see the commonalities and differences on how SPA and CPA are applied.

It revealed that the three textbooks are anchored on spiral progression, even if the chapter on addition has been developed with some differences with regard to the number of lessons and the materials used in teaching activities. Concretely, Singaporean and Japanese textbooks prioritize an in-depth teaching of a relatively limited set of contents, with a variety of materials and techniques. Meanwhile, Senegalese textbooks focus more on breadth than a deep teaching, some contents being overstressed, others such as addition of three numbers not dealt with in the two grades.

This study would likely help to reconsider the content organization in case of curriculum and textbook revision, especially for the newly generalized curriculum in Senegal.

Keywords: textbook, addition, spiral progression approach, pictorial and abstract, curriculum

1. Introduction and background

The necessity to examine the content organization in textbooks is justified by the fact that it is a relatively neglected area (Harden & Stamper 1999). And yet, the quality of textbook has a direct impact on the effectiveness of learning. Henceforth, the acquisition

of new competences labelled 21st century skills requires well-designed textbooks, totally aligned with core ideas defined in the intended curriculum. Today more than ever, the general tendency is to distribute and articulate contents in and across the grades and to break down the barriers between subjects so as to facilitate the mobilization of knowledge and skills. In

that very context, Senegal, Japan and Singapore have adopted curricula anchored in spiral structure.

The definition of spiral curriculum or spiral structure of curriculum is related to Bruner's assumption when he argues that "any subject can be taught in some intellectually honest form to any child at any stage of development" (Bruner, 1977, p.33). In other words, the success or failure of the teaching and learning activities closely depend on teachers' instructional methods and materials since all children are able to understand even sophisticated subject contents. Among others, the Spiral Progression Approach (SPA)¹ has been ascribed many advantages.

First and foremost, concepts are introduced intuitively during the first lessons of grade one. Then, topics, themes or subjects are revisited many times during the learners' cursus. (Harden & Stamper, 1999). However, it is worth mentioning that the review is not a simple task of repetition of the knowledge. It aims at solidifying the prior skill before introducing it in a higher level. In that sense, the spiral curriculum enables a **logical progression**. In other words, curriculum designers, textbook writers and teachers break down the complex ideas into more simplistic chunk of information. The teacher is then requested to help the learner make meaningful connections between these ideas since lessons are just indivisible parts of the same reality and sequence is operated only for teaching purpose.

Another pivotal pillar of spiral curriculum is that basic ideas should be incrementally dealt with in depth in and across the grade through concrete activities, manipulatives and so on. This prompted Bruner to assert that "we opt for in depth and continuity in our teaching rather than coverage" (Bruner, 1979, p.109). Concretely, many textbooks prioritize the teaching of a limited set of contents in order to secure enough time to visit them in depth through various approaches and materials. This means, what we lose on surface (quantity), we catch it up in depth (quality). This idea is summarized by these three words "less is more" for lower graders.

The vertical and horizontal articulation of contents is also another key feature of spiral curriculum. According to Samala (2018), vertical

articulation logically unifies one lesson to another while horizontal articulation connects the knowledge and skill to other disciplines. In the domain of mathematics for instance, a lesson developed in numeracy during a given period of time should logically be connected to contents in measurement and geometry. Ideally, we cannot deepen, for example, the relation between kilogram and gram in measurement if students have not been taught, in numeracy, the number 1,000.

As for the teaching strategies, Bruner advocates **learning through enquiry and problem solving**. This idea is in line with the constructivist approach which fosters learners' active investigation. In that perspective, assessment is meant to give the evidence of sound mastery and teachers should make sure the students really understand previous knowledge before introducing a new one. The basic idea of "broken spiral" developed by Orale (2018) refers to the discontinuity in the learning and acquisition of core competences when students fail to master key skills in the global process.

Moreover, learning as a **shared activity** is an important recommendation made by Bruner (1996) when he advocates an "interactive process in which people learn from each other."

In sum, SPA lays emphasis on the importance of well-articulated contents that are deeply covered through meaningful and concrete activities in order to assure a smooth progression in learning. It is very often combined with the Concrete, Pictorial and Abstract (CPA) approach in lower graders' textbooks.

With regard to lower graders' age, CPA has received a wide applaud in the instructional activities. In some other literature, CPA is also termed Concrete-Representational-Abstract (CRA) or Concrete-Semi-Concrete-Abstract (CSA). It is based on Bruner's belief that human cognition goes, in a discrete way, through the three different but interwoven stages that are enactive, iconic and symbolic. In sharp contrast to Piaget's cognitive development theory, Bruner thinks young children are able to learn many things, as long as the instruction and scaffolding are well-arranged. In that sense, he strongly suggests the representation through "enactive, iconic and symbolic".

¹ SPA means dealing with the same content in and across the grade by gradually increasing the complexity. In that way, there is also a coherent link between lessons and the previous knowledge is repeatedly revisited.

Table 1. Correspondence enactive, iconic and symbolic into CPA.

Bruner's idea	Application on teaching mathematics
Enactive representation: action-based. Thinking is essentially based on physical actions that enable an encoding of information. In short, children learn by doing.	Concrete: Teachers habitually use manipulatives and hands-on to introduce new knowledge intuitively.
Iconic representation: image-based	Pictorial: To support the verbal explanations, teachers use diagrams, visualizations and so on.
Symbolic representation: language-based. Use of codes, symbols. It is acquired around 6 years.	Abstract: Use of words and mathematics symbols (numbers, operation sign and so on.)

Source: adapted from Hui, C. S., Hoe, L. N., & Lee, K. P. (2017)

This approach has been and is still applied by many mathematic teachers throughout the world. Table 1 summarizes the correspondence between the ideas of enactive, iconic and symbolic with CPA.

Mathematics are abstraction in essence. This approach stipulates an increased exposure of the mathematical idea through various materials and the corresponding perceptual pictures and symbols. At the long run, students finally grasp the abstract mathematical idea and the storage is facilitated by the creation of mental pictures.

Matsumoto (2017) rephrases the concept of multiplicity of representation by stressing the necessity to teach from the concrete to the abstract passing by the semi concrete. Since its inception about forty years ago in Singapore, CPA sequence is the key instructional strategy for the development of primary mathematic concepts (MOE, 2007; 2012). In Japan, the course of study endlessly stresses the need to teach through concrete activities. As for the Senegalese case, it is stipulated that “learning activities should include concrete, semi concrete and abstract representations” (MOE, 2009, p.51)

However, in practice, the original idea of experiencing new concepts through physical activities and games is hindered by the formal organization of teaching in classrooms. Performing such activities is limited by time constraints and classroom setting. As a matter of consequence, the enactive stage suggested by Bruner is not always applied because some practitioners do not perceive the importance of learning through enactive representation. In that sense, Matsumoto (2017, p.139) argued that “as we, including children, lack exercise, opportunities to use enactive representation decrease.”

Applied to mathematic teaching for lower graders, CPA implies a process that necessarily starts with genuine physical manipulations of the new

concept to be learned. Then follows a representation with diagrams, drawings just to mention a few. The last part introduces abstract symbols. This approach is advantageous because it changes children’s attitude regarding learning mathematics. The use of manipulatives corresponds to the in-born need of young students to move, see, touch and manipulate. When students learn by doing, they achieve some appreciable results and their self-confidence increases.

2. Objectives and Methodology

2.1. Objectives

Based on the theoretical discussion above about the curriculum structure, this article analyses, on the one hand, the selection and organization of textbook contents in Senegalese, Japanese and Singaporean grade one and two mathematics textbooks, with consideration to Spiral Progression Approach (SPA). The choice of low grades is not done by chance. It is due to the fact that learners in these levels may be easily confused if the content articulation is not well-done. On the other hand, their age requires the use of concrete materials to apprehend and understand abstract notions taught in mathematics.

Moreover, it highlights how the idea of Concrete, Pictorial and Abstract (CPA) is applied in the textbooks of the countries afore mentioned. Likewise, this study compares the three countries cases, shedding a light on similar and/or different traits in terms of SPA and CPA, and any possible lessons among cases to improve content structure of textbooks of each country.

2.2. Methodology

This research prioritizes a document analysis. In fact, a sample of approved textbooks was taken as representative of the country’s textbook key

characteristics. These textbooks were analyzed with respect to SPA and CPA. Since the three countries do not necessarily develop the same contents in the same grade, the research focuses on lessons commonly taught in these levels. In that way, contents related to addition were analyzed because they are learned, with slight differences, within the same range of numbers in the three textbooks.

With respect to SPA, central issues analyzed in the textbook are the number of lessons, pages devoted to the same lesson on addition, the average page of a lesson, the shortest and longest lesson on addition. Furthermore, the analysis investigated how other traits of SPA are applied in the textbook. Concretely, the following issues were considered:

- How the prior knowledge is linked to the new one between lessons, units and grades?
- How consistent is the vertical alignment of contents in the textbooks?
- How is the chapter of textbook, whose unit topic is “addition”, broken up into lessons?
- What is the balance between breadth and depth regarding contents taught in the two grades?

As for CPA, it considers the physical features of textbooks before listing and describing the different materials used to introduce the concept. Moreover, lower graders effectively learn through a smooth process that moves from concrete to abstract, passing through semi-concrete. That is why textbooks were scrutinized to find out how abstract mathematics concepts are introduced, with respect to learners’ psycho-pedagogical status.

3. Results and Discussion

3.1. Application of the Spiral Curriculum Approach

The application of SPA in the textbook is analyzed with regard to the following characteristics: the linkage between previous and new knowledge, the vertical articulation, the meaning of addition, the number of digits involved, the breaking up of big ideas into chunks of knowledge or lessons and the breadth and depth of lessons. Prior to the detailed analysis of the textbook contents, the volume is illustrated in Figure 1.

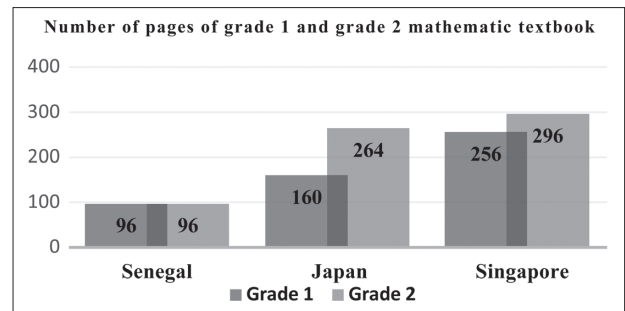


Figure 1. Overview of textbooks.

At first glance, this figure shows that the number of pages of Singaporean textbooks is almost three times bigger than the Senegalese one. In grade two, Singaporean textbook has 96 pages more than the Japanese. In Singapore and Japan, there are two textbooks in each grade. Designing two textbooks (A and B) offers the possibility to deal with contents in details. Furthermore, these textbooks are more handleable for pupils who are generally six, seven or eight years old. Chronologically, textbook A develops the first contents in the curriculum and B deals with higher level. The paragraph below considers how the textbook establishes the connection between the previous and new knowledge.

3.1.1. The Linkage between Prior and New Knowledge

As stated in the first part, the progression is based on prior knowledge, which connects cumulatively each lesson to the following one. The pace of learning shifts from known to unknown things, and from simple to complex contents. In the Senegalese grade two textbook, there are lessons exclusively dedicated to the review of operations learned in grade one. Basically, contents related to addition on two single-digit numbers and two and one-digit numbers are taught in order to refresh students’ knowledge about the meaning and technique of addition. In Japan and Singapore, there are no systematic lessons meant to review the addition contents in grade two. The textbook developer banks on the strong foundation laid in grade one and the actualization of prior knowledge at the beginning of each lesson.

In the three countries’ textbooks, a lesson usually starts with a problem the students are expected to solve based on what they have already learned. Shortly before addition is taught, Japanese and Singaporean textbooks introduce the basic idea of hierarchical inclusion and part-part-whole with

number bonds, ten frames and exercises of decomposition without using an operation sign. Intuitively, learners understand for example that five is the sum of three and two, one and four and so on. Figure 2 illustrates an example of number bond.

These activities are developed to prepare the systematic study of addition. Moreover, the same skill required to add two one-digit numbers with carrying can be applied to easily calculate three one-digit numbers. The following figure shows, for instance, how addition contents are articulated in the textbooks.

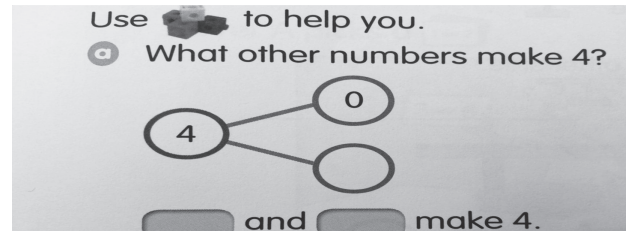


Figure 2. Number Bond.
Source: My Pals Are Here, 1A (2013, p.21)

Country	Decomposition	Addition without carrying	Addition with carrying
Japon			
Singapore			
Senegal			

Figure 3. Sequence of learning contents with respect to “addition”.
Source: Made by the author, based on the three grade one textbooks cited in reference.

In Japanese and Singaporean textbooks, addition is taught with gradual difficulties. The strong linkage between lessons enables students to easily reactivate the previous knowledge in order to deal with more complex contents. For instance, decomposing numbers through games equals, to a certain extent, doing addition without the operation sign. When students effectively know the possible combinations into which a number can be decomposed, they intuitively acquire the meaning of addition: two parts at least composing a whole.

In the Senegalese textbook however, there is tiny misalignment which could result into a cognitive

dissonance or hinder students' understanding. In fact, addition is introduced before decomposition. It is more relevant to teach decomposition first in order to prepare lessons on addition.

3.1.2. Vertical Articulation of Contents.

Vertical articulation of contents on addition considers the meaning, the number of digits involved, the necessity to carry or not and lessons dedicated to consolidate the sense of quantity or preparation for multiplication. Table 2 summarizes the logical structure of contents in the textbooks.

Table 2. Vertical alignment of contents of “Addition” in grade one and two

Country textbook	Organization of lessons on addition
Senegal	Contents related to addition are basically organized as follows: First of all, the meaning of addition as contrasted to subtraction is introduced. Then, addition without and with carrying within 13, 16, 19 and 20 in grade one is dealt with. In grade two, there are some lessons devoted to the review of the meaning and practice of addition with 20, then follows addition without and with carrying within 100.
Japan	Japanese textbooks first introduce the meaning of addition. It has two components: addition as combining two numbers and addition as an increase of an initial number. Students are also taught how to write math sentence. Subsequently, there are lessons on addition involving two and one-digit numbers, three one-digit numbers, carrying, addition of zero, addition of the same number within 100. In grade two, addition with carrying (two digits plus one-digit number) is taught. It is followed by lessons on vertical calculations, addition with tens as a unit (30+80 for example) and finally addition of three numbers and commutativity.
Singapore	In grade one, addition within 20, 40 and 100 is introduced. It includes writing math sentence and a story problem and uses different materials and techniques of calculation such as number bond, counting on, making ten. Addition without and with regrouping is also taught, following the same range of numbers. Vertical calculation is introduced in grade one. In grade two, addition is studied within 1,000.

With respect to the necessity to arrange the contents relevantly, lessons on addition in the three countries have been aligned with the study of numbers. In other terms, the operation is learned within the range of numbers studied so far. In Senegal students learn numbers from 0 to 100 whereas Singaporean and Japanese learners' study up to 10,000 (ichi man). In general, selected contents taught in these textbooks are vertically well-aligned. Difficulties are gradually introduced and each prior knowledge meaningfully connects to the following one.

Nevertheless, in the Senegalese textbooks, the omission of some key lessons such as addition of three-digit numbers and making a story problem based on a math sentence hinder the understanding of this operation. In fact, addition is not limited to two numbers. In daily situations, students regularly encounter situations in which they add up three numbers.

3.1.3. The meaning of addition

The other similarity is that, in the three countries, the meaning of operations precedes the technique of calculation. Japanese textbooks clearly differentiate two meanings: addition as combination of two quantities (number) and addition as an increase of an initial number.

However, the Senegalese textbook devotes special lessons on the meaning of addition, by introducing it as the inverse operation of subtraction at the very

beginning of grade one. These lessons are repeated in grade two. Students are given at random many story problems on addition and subtraction, and they are expected to identify the appropriate operation for each case.

At the contrary, Japanese and Singaporean textbooks combine the teaching of meaning with the technique of calculation. The story problem introduced at the very beginning of the lesson requires an interpretation, a writing and justification of a math sentence. Moreover, these two countries' textbooks also develop lessons on making addition story problem. Learners should be fluent to move from the math sentence to the story problem and vice versa.

3.1.4. The Number of Digits Involved

The number of digits and carrying are salient criteria on which the gradual deepening of lessons on addition is based. Almost in all textbooks, students learn addition of two one-digit numbers without and with carrying. However, in Japanese and Singaporean textbooks, there are lessons on three one-digit numbers that help introduce the commutativity of addition, named order of calculation. Then, it is followed by addition of two and one-digit numbers without and with carrying. Vertical calculation is introduced in grade one in Singapore in the first lesson of book B whereas Japanese and Senegalese textbooks plan this lesson in grade two.

Moreover, lessons meant to deepen sense of

Table 3. Coverage of “Addition” contents in the three countries’ textbook

Countries	Total lessons on addition		Total pages of lessons on addition		Average page per lesson		Number of pages of the longest lesson on addition		Number of pages of the shortest lesson on addition	
	Grade 1	Grade 2	Grade 1	Grade 2	Grade 1	Grade 2	Grade 1	Grade 2	Grade 1	Grade 2
Senegal	4	8 (with 3 lessons of review)	4	10	1	1	1	1	1	1
Japan	10	8	22	22	2.2	2.75	6	5	2	2
Singapore	10	11	36	35	3.3	3.18	5	6	2	2

quantity and the decimal notation system are also designed in Japanese and Singaporean textbooks. For example, adding zero highlights the quantity of this number as students find out that the result does not change, so there is no increase. Likewise, Japanese and Singaporean textbooks teach addition that uses ten as a unit, for example $20+70$. This is a strong way to illustrate the idea of position value. Finally, adding the same number (repeated addition) is taught not only to apply the technique learned before, but also to prepare the teaching of multiplication.

3.1.5. Breaking up Big Ideas into Chunks of Knowledge or Lessons

The structure described above applies the idea of progressing from simple to complex contents. The unit contents of “Addition” itself is a broad chapter that encompasses addition of different numbers (integers, decimals, fractions), different digits, implying carrying or not. Grade one and two basically focus on addition of integers from 0 to 100 in Senegal and Singapore. In Japan, even if students learn numbers up to 10,000, the operation rarely goes beyond 500. In the three countries’ textbooks, each lesson deals with simple content. Decimal numbers and fractions are generally learned in upper grades.

3.1.6. Breadth and Depth of Lessons: “Less is More”

The contents related to addition are covered in breadth and depth across the grades. In the first two grades of elementary school, the focus is laid on addition of integers. The table below retraces how they are distributed in the textbook.

First of all, it is worth mentioning that lessons on review and exercises of consolidation are not taken into account in this analysis because countries generally design corresponding workbooks. As we

can notice, the number of lessons devoted to addition in Japan and Singapore are the double in Senegal. Similarly, the total number of pages dedicated to addition in grade one textbook is 22 in Japan and 36 in Singapore whereas in Senegal it is only 4. Striking the balance between lessons on numbers and operations is a crucial issue in mathematics textbooks. In Senegal, a simple counting of lessons reveals that contents on numbers outnumber by far lessons on operations. So, the teaching rarely goes beyond the simple acquisition-application of rules. In that way, addition with carrying is considered as the same technique since there is no gradual differentiation based on the number of digit and the position of carrying in ones, tens and hundreds.

Conversely, Singapore and Japan have taken the option to cover a limited number of contents and to explore them deeply: “less is more”. In other terms, the lessons covered in these grades are deeply visited to lay a solid foundation for future studies. In grade two, the average number of pages for a lesson is only 1 in Senegal, 2.75 and 3.18 respectively in Japan and Singapore. Japanese and Singaporean textbooks develop lessons in many pages so as to encompass a high variety of activities.

To help learners master a concept, it is highly recommended to diversify the situation and the material. In that respect, Singaporean grade one textbook introduces different ways to add. The differentiation goes beyond the sole range of numbers involved and whether or not there is carrying. It encompasses the position of carrying (in one and tens) and the material used to teach. By introducing the same contents in varying the material and focal point, the expected result is to have a deep understanding instead of simply acquiring rules and formula to apply afterwards.

Furthermore, SPA also insists on the importance

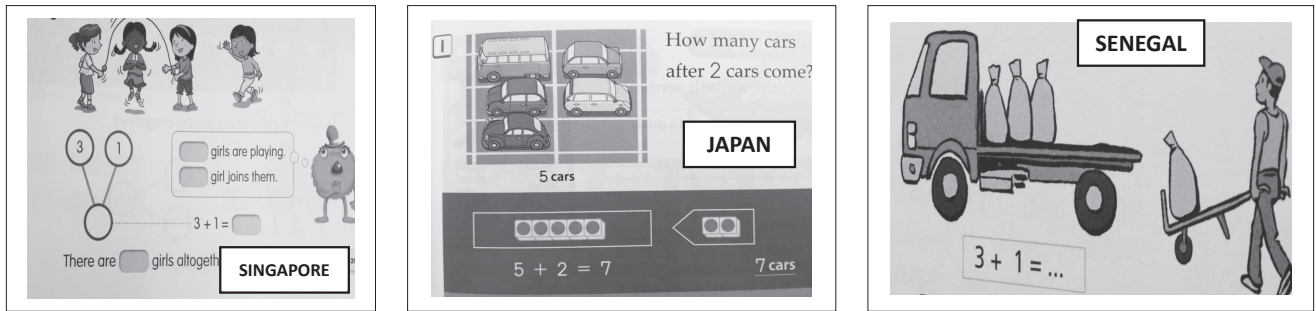


Figure 4: Illustration of CPA in grade 1 textbook. Source: My Pals Are Here, 1A (2013, p.33), Fun with Math 1 (2011, p.41), Mathématiques, Manuel de l'élève au CI (2015, p. 34).

of connecting meaningfully scattered chunks of knowledge. This requirement is applied even though the terms used are slightly different. In Senegal, there are activities of “integration” which expect the student to mobilize prior knowledge to solve problems of higher levels. In the Japanese textbooks, “putting your knowledge to work” are activities meant to help students activate their knowledge in order to consolidate their aptitudes as problem solvers. In the Singaporean textbooks, exercises termed “put on your thinking cap” challenge students to solve non-routine questions by putting to work appropriate heuristics and logical thinking.

3.2. Application of the Concrete, Pictorial and Abstract Approach

Figure 4 illustrates how “Addition” contents are presented in grade one respectively in Singapore, Japan and Senegal.

These pictures illustrate how lessons on addition of two one-digit-numbers are introduced in grade one textbooks. They all show an appreciable application of CPA: lessons start with concrete activities and close with abstract representations after the semi-concrete step.

Most of the time, the textbook developers present characters or situation students are familiar with. In the Singaporean case, we find students playing during

recess. Likewise, Japanese students see almost every day cars parking next to others and, finally, Senegalese students are also acquainted with harvest being transported.

However, there are slight differences in the way this psycho-pedagogical requirement is applied in the textbooks. In the Senegalese textbook, the semi-concrete representation is missing. The concrete phase is directly associated with the abstract representation of numbers. Singaporean students are expected to use cubes and write the addition story whereas Japanese ones must find the total number of cars if 2 more arrive.

Elsewhere, there is a controversy about the notion of concrete activity. In the three textbooks, it refers to visual objects students can touch, see and manipulate. However, some specialists think concrete (enactive) would be conducted, as much as possible, especially in grade one, through activities involving students’ movement, actions and manipulation, sometimes in form of games. Table 4 lists the concrete teaching materials presented in the textbooks.

The basic idea is to introduce abstract notions with concrete materials. The most striking difference is the variety of materials used to introduce the same contents. Ten frames are precious materials to teach both numbers and operations because the quantity they represent can easily be subitized.

Table 4. List of concrete materials designed in the textbook to teach numbers and operations.

Countries	Teaching materials
Senegal	Sticks, bottle caps, stones, place value chart.
Japan	Ten frames, number cards, flashcards, sticks (hundreds, tens, ones), coins and fake notes of 1,000 and 10,000, diagram, number line, bar models.
Singapore	number bond, interlocking cubes, base ten materials, number cards, number tape (count on, count back), ten frames, place value chart, dice (regrouping), stick, coins, fake notes, bar models.

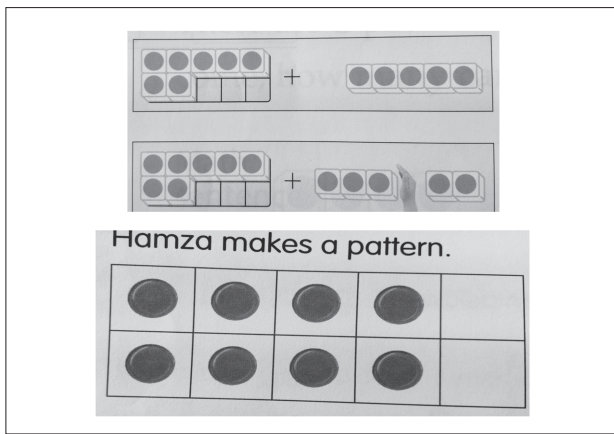


Figure 5: Ten frames in textbooks. Source: Fun with Math 1 (2011, p.80), My Pals Are Here, 1A (2013, p.16),

The importance of diversifying the material and situation can also be justified by the fact that “there exists a multiple of intelligences, quite dependent of each other; that each intelligence has its own strengths and constraints” (Gardner, 1999, p. xxxvii). So, the aptitude to perceive objects is unequally developed from one student to another. For example, some learners may be comfortable with bar models, and ill at ease when we use other materials. Children learn differently. As a matter of consequence, focusing only on linguistic and logical-mathematical intelligences would hinder other students’ aptitude to learn. In a nutshell, the more the material and situations are diversified, the more the chance for students to understand. Beyond the objectives related to the acquisition of knowledge and skill, hands-on materials/ activities help develop dexterity.

In addition, CPA is rooted on the principles of perceptual variability in these textbooks. In fact, (Sriraman, 2007, p.132) confirms Zoltan Paul Dienes’ idea of the multiple embodiment principle by arguing that it is very important to use diverse didactic mathematical tools. He states:

“For example, Dienes placed a strong focus on multiple embodiments and on cyclic patterns of learning where students progress from concrete to symbolic formats in developing an understanding of mathematical structures.”

When the materials used are not diversified, the learner cannot correctly do the abstraction because he is unable to decenter and separate from that material. The nature, shape and structure of the

material introduced in order to illustrate concepts cannot be done at random. It is judiciously planned: first proportional material such as blocks, then card with figures and finally, working without these materials.

The physical characteristics of textbooks are important. In fact, pictograms are part and parcel of textbook cohesive organization as they clearly indicate, in a suggestive way, what action to perform. In other terms, they represent the different key points of the process of teaching and learning. In the Japanese and Singaporean textbook, they indicate, when to ask interpretative questions, suggest a task or summarize main ideas. In the Senegalese textbook under consideration, there are no pictograms.

The main characters of the three textbooks are children with famous nouns. This can considerably enhance students’ interest because the situation depicted deals with their daily life. The gender balance is also respected by the textbook developer. In addition, there is a specific pictogram that is meant to organize the school-parents communication. It expands the lesson by indicating appropriate tasks or exercises parents could do to support the learners.

4. Conclusion and Recommendations

This current article analyzed the content structure in mathematics textbooks for lower graders’ in Senegal, Japan and Singapore. The main purpose was, on the one hand, to find out how SPA and CPA are applied. On the other hand, the commonalities and differences are considered in the three textbooks. The importance to design coherent content organization relates to the necessity to teach from simple to complex knowledge. In terms of methodology, the key traits of these approaches are recalled and samples of approved textbooks were analyzed with regard to these two approaches.

It has been found that, in general, the basic ideas of SPA have been well-applied in the three country textbooks under consideration. First of all, the connection between prior and new knowledge is usually well-established between units and grades. Indeed, there are periodical non routine exercises that demand the mobilization of previous knowledge to solve challenging problems. However, some tiny elements need to be reconsidered in the Senegalese textbook. In fact, lessons on decomposition are

developed after the systematic study of addition: this procedure does not really facilitate the understanding of the meaning of addition.

As for the vertical articulation, it has also been found that lessons are well-phased in and through the grades. In sum, the sequence starts from simple to more complex contents in the textbooks. Regarding the chapter of addition in lower grades, the basic lessons generally studied in curricula throughout the world, in application of SPA, are: meaning of addition and technique of calculation involving addition without and with carrying. In that sense, the organization adopted in the Japanese, Senegalese and Singaporean textbooks is coherent.

However, there is a difference in the option between breadth and depth. Japanese and Singaporean textbooks have studied addition in depth using different techniques and materials such as make ten, which is not the case of the Senegalese textbook which focuses, most of the time, on the acquisition and application of rules, namely the technique of carrying.

As for CPA, this study reveals that much attention has been paid to align the textbook contents to the basic ideas fostered by this approach. In that sense, the three textbooks use appropriate visualizations of core ideas of the lesson, from the concrete to the abstract, passing through semi concrete. For instance, ten frames are widely used in Japanese textbooks because, among other reasons, they enable students to easily recognize numbers and to do calculations, moving freely from ones to tens and vice versa. Likewise, in Singapore, number bonds are frequently used during the first lessons of grade one to decompose and do addition without and with the operation sign. Senegalese textbooks also use concrete representation to introduce addition, even though there are longer and more story problems, with less visualization and representations as compared to the other countries.

In terms of differences, the Japanese and Singaporean textbooks use more materials and techniques to teach the same content. In short, the principle of perceptual variability fostered by this approach is more salient in the two Asian country textbooks than in the Senegalese one.

Based on the above-mentioned discussion and findings, it is suggested to consider the implementation of the following measures, depending on each country's

specificity:

Senegalese textbook:

- ✓ Cover deeply the chapter on "Addition" by aggregating lessons on addition of three numbers and commutativity, addition with ten as a unit, addition of zero to consolidate the understanding of that number.
- ✓ Reconsider the application of CPA by improving the number and size of visualization by clearly focusing on the three parts: concrete, pictorial and abstract.
- ✓ If the financial resources are available, it is preferable to design two textbooks so as to teach contents in details, incorporate bigger and more illustrations and facilitate the reading and handling of this material.
- ✓ Add suggestive pictograms in order to indicate the different steps in the lesson process: reading, interpretation of the story problem, summary of key points and so on.
- ✓ Strike a balance between numbers and operations by adding more lessons on addition for example.
- ✓

Japanese textbook

- ✓ Design a reteach booklet for struggling students for an additional support. They can include pictorial models and many worksheets on non-mastered knowledge items.
- ✓ On the other hand, excelling students can also be provided with more challenging exercises.
- ✓

Singaporean textbook

- ✓ In addition to existing materials, using the ten frames, especially in the first lessons on numbers and addition would be very helpful to deepen the understanding.

References

- Bruner, J.S. (1977). *The process of education*. Harvard University Press, p.33.
- Bruner, J.S. (1979). *On knowing: Essays for the left hand*. Harvard University Press, p.109.
- Bruner, J. S. (1996) *The culture of education*. Harvard University Press, p.22.
- Fun with math 1*. (2011). Shinko Shuppansha Keirikan.

- Japan, pp. 16-80.
- Fun with math 2 A.* (2011). Shinko Shuppansha Keirikan. Japan, pp.19-21.
- Fun with math 2 B.* (2011). Shinko Shuppansha Keirikan. Japan, p.76.
- Gardner, H. E. (2011). *The Frames of mind: The theory of multiple intelligences*. Basic Books. New York, p. xxxvii.
- Harden, R.M., & Stamper, N. (1999). What is spiral curriculum? *Medical Teacher*, 21(2), pp.141-143.
- Hui, C. S., Hoe, L., N., & Lee, K., P. (2017). Teaching and learning with concrete-pictorial-abstract sequence: A proposed model. *The Mathematics Educator*, 17(1), pp.1-28.
- Ka, M., Faye, B., Kassé, C. & Gaye, A. (2015), *Mathématiques : Manuel de l'élève au CI. (Mathematics: Student's book: Grade one)*. Hachette Livre International, pp.34-35.
- Ka, M., Faye, B., Kassé, C. & Gaye, A. (2015), *Mathématiques : Manuel de l'élève CP. (Mathematics: Student's book: Grade two)*. Hachette Livre International, pp.24-25.
- Keong, F., H. & Wah., B., L., P. (2013). *My pals are here: Maths 1A (3rd ed.)*. Singapore. Marshall Cavendish, pp.16-51.
- Keong, F., H. & Wah., B., L., P. (2013). *My pals are here: Maths 1B (3rd ed.)*. Singapore. Marshall Cavendish, pp. 17-22.
- Keong, F., H. & Wah., B., L., P. (2013). *My pals are here: Maths 2A (3rd ed.)*. Singapore. Marshall Cavendish, pp. 22-41.
- Keong, F., H. & Wah., B., L., P. (2013). *My pals are here: Maths 2B (3rd ed.)*. Singapore. Marshall Cavendish, p. 22.
- Ministry of Education of Senegal. (2009). Fascicule de didactique des disciplines à l'élémentaire (Booklet of subjects taught at elementary school level). Dakar, p.51.
- Matsumoto, K. A. (2017). Review of Jerome Bruner's education theory: Its implication for studies in teaching and learning and active learning. *Journal of Nagoya Gakuin University*, 53(4), pp.129-146.
- Orale, R. (2018). When the spiral is broken: Problem analysis in the implementation of spiral progression approach in teaching mathematics. *Journal of Academic research*, 3(3), pp.14-24.
- Samala, H., L. (2018). Spiral progression approach in teaching science: A Case study. *4th Research Conference on Higher Education*, (2018), pp.555-567.
- Sriraman, B. (2007). *Zoltan Paul Dienes and the dynamics of mathematical learning. The Montana Mathematics Enthusiast*. Montana. USA, p.132.