# Teachers' Content Knowledge and Pedagogical Content Knowledge for Teaching: As Preconditions to Develop Students' Mathematical Thinking at Grade 1-3 in Nepal 

Reetu SHRESTHA<br>Global Education Course, Graduate School, Naruto University of Education


#### Abstract

Teachers' content knowledge and pedagogical content knowledge defined by Ball et al. (2008) and Shulman (1986) have been used as a conceptual framework to investigate teachers' mathematical knowledge for teaching at grade 1-3 based on new developed teaching materials of Nepal. Online based questionnaire survey was applied for gathering the data and analyzed by using 3 -scale rubrics. This study focused on teachers' content knowledge (CK) and pedagogical content knowledge (PCK) on number sense and basic operation. From overall analysis of data, researcher argues that teacher's CK and PCK are not sufficient to use new materials appropriately. So, it is suggested that professional development support for teachers is seems to be necessary in order to improve teachers' CK and PCK.


Keywords: Teachers' mathematical knowledge for teaching, content knowledge, pedagogical content knowledge, Students' performance, new teaching and learning materials

## 1. Introduction

High failure and under achievement have been a perennial problem in school education of Nepal. Educational Review Office (ERO) conducts National Assessment of Student Achievement (NASA) at different years. The report of NASA has showed the average of students' achievement in mathematics is 60 \& 45 at grade 3 in 2012 \& 2015 respectively. Furthermore, ERO has conducted National Assessment of Reading and Numeracy (NARN) in 2021 with intending to identify the current status of Grade 3 students' reading and numeracy skills. According to the report of NARN, average of student's achievement in numeracy is $37.22 \%$. Students' achievement in numeracy was calculated by taking sub-tasks as geometrical shapes identification, identification of angle and side, number identification, use of arithmetic signs, fraction, clock reading and daily life mathematics. The
report revealed that number identification, use of arithmetic signs subtasks are seem to be challenging for the students (ERO, 2020).

The below Figure 1, shows the students' achievement is below than minimum national standard, $-60 \%$ (DoE, 2017) and is also getting low instead of increasing at grade 3.


Figure 1. Trends of student achievement in mathematics at grade 3
Source: Developed by researcher, based on ERO $(2018,2020)$

On the other hand，Japanese project named ＇Improvement of Mathematics Education in Nepal （IMEN）＇is lunched with technical support from Japan International Co－operation Agencies（JICA）in Nepal from 2019．Now，Ministry of Education，Science and Technology（MoEST）and IMEN are collaboratively working together with the goal to improve the foundational Mathematics proficiency of primary school students．For improving the foundation or to enhance students＇mathematics performance，MoEST and IMEN have the strategy that to develop quality curriculum，textbook and teacher＇s guide；then support to the teachers through effective professional development training from Center for Educational Human Resource Development（CEHRD），Educational Training Center（ETC）of province and local level agencies；also，to develop effective mechanism for managerial support with monitoring and supervision． Accordingly new curriculum，student＇s workbook and teacher＇s guide have been developed and now these new curricular materials are under the implementation．

New materials are the means of teaching and learning．Appropriate implementation of new materials depends upon the effectiveness of teaching． However，teaching in many places and cases has not been as effective as expected．Especially，there are four factors：student，teacher，school management， and family background（Suhaini et al．，2020）that affect directly and indirectly to effectiveness of teaching and learning as well as students＇learning performance． Student factor includes individual differences，gender， motivation or attitudes etc．Similarly，teacher factor refers teachers＇knowledge for teaching，teaching experience，attitude，belief，motivation and professional development training．Family factor refers to family background like socio－economic or parents＇education and family environment like parental time，educational environment etc．Likewise，school factor refers school leadership，physical facilities and school environment． Among them，factors related to teacher deserve high significance because none of the other elements can function well while the teacher element remains weak and ineffective，which paralyzes the whole education system（Rijal，2016）．

Moreover，effective teaching requires teachers to have specific knowledge and skills．Shulman（1986） suggested that person knows something does not mean that this person can teach this issue．He claimed
that teaching necessarily begins with a teacher＇s understanding of what is to be learned and how it is to be taught．Similarly，Ball，Thomes \＆Phelps（2008） argues that teachers who do not themselves know a subject well are not likely to have the knowledge they need to help students learn in this content and also teacher need to know the ways that useful for making mathematical sense of student work and choosing powerful ways of representing the subject．

From this discussion it is clear that teacher should be competent on both content knowledge（CK）and pedagogical content knowledge（PCK）．Shulman（1986） defined CK as amount and organization of knowledge per se in the mind of the teacher and PCK as a second kind of knowledge that goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching．

The general objective of this research study is to find the way for proper application of new materials in order to develop student＇s mathematical thinking． This research study will focus on exploring teachers＇ CK and PCK for teaching mathematics at grade 1－3 based on new curricular materials in order to apply these materials appropriately．Thus the research question of the study is as follows：
－How is teacher＇s content knowledge（CK）and pedagogical content knowledge（PCK）for teaching at grade 1－3 in Nepal？

## 2．Literature Review

## 2．1 Teachers＇CK and PCK

The term CK and PCK was introduced into the discourse of teacher education in American Educational Research Association written by Shulman （1986）．He pointed out that mathematics teachers should have three different knowledge；content knowledge，pedagogical content knowledge and curriculum knowledge．That was great excitement for the researcher and after that many researcher have given different view towards teachers＇ knowledge．

In an elaboration of Shulman＇s（1986）notion of pedagogical content knowledge several research teams（Ball et al．，2008；Hill et al．，2008）have developed the domain of content knowledge for teaching into two initial categories of Shulmans＇teachers＇knowledge for teaching as subject matter knowledge and
pedagogical content knowledge as shown in Figure 2.


Figure 2. Mathematical Knowledge for teaching model Source: Ball et al., (2008, p.403.)

Content Knowledge (CK) includes three subdomains: common content knowledge (CCK), specialized content knowledge (SCK) and knowledge at mathematical horizon (KMH). CCK refers mathematical knowledge and skill used in general setting other than teaching (Hurrell, 2013). It includes that teacher should be able to calculate or solve mathematical problems correctly, use terms and notation correctly, and recognize wrong answer and an inaccurate definition of the textbook if mentioned. In addition, SCK refers mathematical skills and knowledge particular to teaching (Hurrell, 2013)¹. Similarly, KMH is an awareness of how mathematics included in the curriculum (Ball et al., 2008). It includes the vision useful to see connections among various mathematical ideas. This knowledge refers connection of mathematical topics, make connections between the different strands in mathematics (Hurrell, 2013). Else, KMH measures teacher's awareness of prerequisite knowledge that is required to reinforce students' concept formation (Khakasa \& Berger, 2016).

Likewise, Pedagogical Content Knowledge (PCK) includes three sub-domains: knowledge of student (KS), knowledge of pedagogy (KP) and knowledge of curriculum (KC). KS is combination of knowing about student and about mathematics (Hurrell, 2013). Teacher must anticipate what students are likely to think and what they will find confusing (Ball et al., 2008) or difficult or easy when completing a task. Even more, teacher must recognize and articulate misconceptions of students and be familiar with students and their mathematical thinking. In contrast,

KP is combination of knowing about teaching and mathematics. This knowledge allows teachers to identify sequence of particular content for instruction, evaluate advantages and disadvantages of different examples and representation for content, and also to identify methods and procedures afford instructionally (Ball et al., 2008). KC refers the knowledge of curriculum includes selecting and using suitable curriculum materials, fully understanding the goals and key ideas of textbook and curricula (NCTM, 2000).

In this study, the researcher has followed the concept of both researcher teams: Shulman (1986) and Ball et al. (2008), and used the model of mathematical knowledge for teaching based on two knowledge domains: CK and PCK. CK includes what to know, why it is worth to know and how to apply it. It refers the knowledge related to calculate and solve mathematical problem, use mathematical notation; justify mathematical expression or give reason; and make connection with mathematical content or other discipline. Similarly, PCK includes curriculum knowledge, knowledge of pedagogy, and knowledge of student. Curriculum knowledge refers understanding and articulating the goals, key ideas of textbook and curricula. Similarly, knowledge of pedagogy refers representation of the content, instructional Strategies and knowledge of student includes understanding student's thinking, misconception, assess and provide feedback.

### 2.2 Conceptual Framework

There is triangular relation among three components; student, teacher and developed materials in pedagogical situation. They are interrelated to each other. Students and content are linked through learning, study \& work; teacher and content are linked through preparation \& instruction; teacher and students are connected through the much-studied student-teacher relationship or pedagogical relation; and, a teacher relates to the student through the content (Friesen, 2017). Teachers' content knowledge and pedagogical content knowledge for mathematics teaching influenced the relation of student and content in this pedagogical situation. Here, CK and PCK are dependent variable which can lead to the effective

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Figure 3．Conceptual framework adapted from conceptual model of the Friesen（2017）and Desimone（2009）． Source：developed by researcher
teaching if the teachers＇level of proficiency in CK and PCK and instruction is improved．Professional development support is one of the suitable options to enhance the teachers＇proficiency level of CK and PCK．Teacher professional development is considered as a valuable strategy to strengthen teachers＇content knowledge and pedagogical content knowledge and thereby promotes and improves classroom teaching practices（Desimone，2009；Fischman \＆Riggs，2021； Matos el at．，2009）．Finally effective classroom teaching improves the students＇learning in mathematics which helps to develop mathematical thinking among student．So the baseline information about teachers＇ CK and PCK is very important．

The Figure 3 shows the conceptual framework for this study．

## 3．Methodology

## 3．1．Research Design

The Research design of the study was based on both quantitative and qualitative research methods． 99 teachers who teach mathematics at grade 1－3 were selected through purposive sampling method from Karnali，Gandaki and Bagmati province of Nepal in order to explore teachers＇CK and PCK．

## 3．2．Research Instrument

A questionnaire survey was administered through Google Forms to collect the information for teachers who teach mathematics at grade 1－3 through Google Forms．Based on the report of NARN ${ }^{2}$ ，number sense and basic operation were taken for content of the study．As Table 1 shows，the questionnaire consisted of two parts：background information and teachers＇ knowledge．Background information part had included 5 questions related to teachers＇teaching experience

Table 1．Item＇s detail

| Item description | No．of items | types of items | Content |
| :--- | :---: | :--- | :--- |
| Background <br> information | 5 | Multiple－choices | Teaching experience，participation on training |
| Teachers＇Knowledge |  |  | Then |
| Content knowledge | 6 | Fill in the blank／ <br> multiple choice and <br> open－ended | Comparison of greatest and smallest number by making <br> of 4 digits number；meaning of multiplication；relation <br> of multiplication and division；and decomposition and <br> composition of number． |
| Pedagogical Content <br> Knowledge | 10 | multiple choice and <br> open－ended | Teaching activities and materials for number 10；content <br> sequence of subtraction；importance of problem solving and <br> CPA；understanding of students＇thinking． |
| Total | 21 |  |  |

Source：Developed by researcher

[^1]and participation status in teacher training during the past three years. Similarly, teachers' knowledge part had included 6 questions related to CK and 20 questions related to PCK. Items were included both multiple choice and open-ended types of questions.

### 3.3. Data Analysis

Researcher had been developed 3-point scale rubrics to quantify teachers' written responses for analysis (see in Table 2). According to the rubric, a numeric value ( 0 to 2 ) was assigned to each teacher's response. The rubric was designed to incorporate all possible responses expected from the given item with a detailed rating scale ranging from correct response, partial correct response and incorrect response. For each perfect or all correct responses with clear and appropriate explanation was awarded as 2 scores; partially correct responses was awarded as scores rating 1 ; and incorrect response, or incoherent response, or no response was awarded as 0 score based on the rubric description. Especially 9 items were quantified on the basis of this rubric and 7 items were analyzed qualitatively.

Table 2. General 3-point scale Rubric

| Score | Description |
| :--- | :--- |
| 2 | All correct responses, or correct response <br> with clear and appropriate explanation |
| 1 | Correct response with not clear explanation, or <br> Partial correct response with explanation, or |
| 0 | Partial correct response without explanation <br> incorrect response, or incoherent response, or <br> no response |

Source: Developed by researcher based on Khakasa \& Berger (2016)

## 4. Findings and Results

### 4.1 General information of participants

Especially the teacher participants were selected those who are teaching Mathematics at grade 1-3. Table 3 shows the general information of participants. The gathered data have shown that all participants have at least one year teaching experience. Also, more than $60 \%$ teachers have at least 5 years teaching experiences. New materials have been implemented from 2020. This means all participants are seemed to have one year experienced about new materials. Similarly, 55\% participants were involved in different
types of teacher training during the past three years. Even more, 48\% of them were involved in the curriculum dissemination workshop for new materials. From this information it suggested that they already knew about the new change.

Table 3. Respondents information

| Description | Total <br> Numbers | Remarks |
| :---: | :---: | :---: |
| No. of participants | 99 | 50 from Karnali, and 49 form Bagmati \& Gandaki province |
| Teaching experiences |  |  |
| $\begin{aligned} & \mathrm{Y}<5 \text { yrs } \\ & 5 \leq \mathrm{Y}<10 \text { yrs } \\ & 10 \leq \mathrm{Y}<15 \text { yrs } \\ & \mathrm{Y} \geq 15 \text { yrs } \end{aligned}$ | $\begin{aligned} & 36 \\ & 26 \\ & 16 \\ & 21 \end{aligned}$ | More than $60 \%$ teachers with at least 5 years teaching experiences |
| Participation on Training |  |  |
| $\begin{aligned} & \text { Yes } \\ & \text { No } \end{aligned}$ | $\begin{aligned} & 54 \\ & 45 \end{aligned}$ | Among the teacher who have training experiences, 26 (i.e $48 \%$ ) participated on curriculum dissemination program |

Source: Developed by researcher

### 4.2 Quantitative Analysis

### 4.2.1 Teachers' CK and PCK

6 items related to CK and 10 items (also including sub-items) related to PCK were asked in questionnaire survey. Among them 4 items of CK about comparison of greatest and smallest number by making of 4 digits number, meaning of multiplication, relation of multiplication and division, and decomposition and composition of number; and 5 items of PCK about content sequence of subtraction, importance of problem solving and Concrete-Pictorial-Abstract (CPA), and understanding of students' thinking were quantified on the basis of 3 -point scale rubric. Figure 4 shows the summary of result that is found from quantitative analysis about teachers' mathematical knowledge in different domain. The findings revealed


Figure 4. Teachers' CK and PCK
Source: Developed by researcher
that average level of respondents＇CK and PCK proficiency are below $40 \%$ for correct responses．

## 4．2．2 Comparison of trained and untrained teachers＇ mathematical knowledge

Among 54 teacher who have training experiences， 26 （i．e．，48\％）were participated on curriculum dissemination workshop of new curricular materials． Here，the researcher had taken respondents who have participated on curriculum dissemination workshop as trained and who have not participated on that workshop as untrained for this statistical comparison． The comparison was done between trained and not trained respondents by using quantified scores through statistical analysis．The Figure 5 shows the Mann－Whitney U－test between trained and untrained respondents at significance level 0.05 with p－value 0.272 ．The finding indicates that there is no significance difference among trained and untrained teachers with mean rank 55.25 and 48.13 respectively．From this finding，researcher concluded that only dissemination workshop and discussed contents on the workshop are not sufficient for teacher to enhance teachers＇ knowledge about new mathematical content and approach that mentioned in new curricular materials．


Figure 5．Comparison of Independent－Samples Mann－ Whitney U Test for trained and untrained teachers＇mathematical knowledge Source：Developed by researcher

## 4．3．Qualitative Analysis

## 4．3．1．Calculation of addition

This item was asked in order to explore teachers＇ knowledge on calculation or solve mathematical problems correctly and explain，or justify one＇s
mathematical ideas．Accordingly，question was asked to calculate $8+7=$ ？，with explanation．Respondents＇ responses were categorized as in Table 4；

## Table 4：Rubric for calculation of addition

| Category | Description |
| :--- | :--- |
| Count all | Starts from number 1 and continues to | count by enumerating each unit

Count on Start from one of the addends and continues to count by enumerating unit（eg．count from 8 seven words）； and（only $7+8=15$ ）
Base－10 Transforms the original problem into decomposition two or more simpler problems using the base 10 properties or the numbers system［eg． $8+7=(8+2)+5=15)$ ］
Fact based Transforms the original problem into decomposition two or more simpler problems using previously memorized number fact［eg． $\left.\left(7^{*} 2+1=15\right)\right]$
Other Strategy cannot be understandable，or only calculation，or not knowing or no answer
Source：Developed by researcher based on E．V．Laski et al．（2014）

The data（Figure 6）revealed that only $2 \%$ of respondent teachers are used the base－10 decomposition strategy ${ }^{3}$ ， $29 \%$ are used to the counting all strategy，and $17 \%$ are used count on strategy to add one digit and one digit number．But 48\％responses were found with not understandable or not knowing strategies．


Figure 6．Strategy for addition of one digit and one digit． Source：Developed by researcher

From the analysis of teachers＇responses it was concluded that they did not use base－10 decomposition strategy．In contrast，new mathematics textbook has mentioned base－10 decomposition strategy to add one digit and one digit number．

[^2]
### 4.3.2. Teaching activities and materials for number 10

This item was asked to write the teaching activities and teaching materials to teach number 10 as a first lesson. The purpose of the item was to find whether teacher have discussed about base -10 structure or not while teaching number 10 . The Figure 7 shows the responses related to teaching activities. It shows that only 5 respondents are discussed base-10 structure like making group of 10 or bundle of 10 after single digit counting ${ }^{4}$. In contrast, $37 \%$ respondents are used only single digit counting strategy. Similarly, $50 \%$ responses were found with not understandable strategy.


Figure 7. Activities for teaching number 10. Source: Developed by researcher

Figure 8 shows the responses related to use teaching materials in teaching number 10 as first lesson. Only $13 \%$ respondents are used base-10 materials like base-ten block and most of the respondents are used local materials like marbles, stones, pencils etc. but they have not mentioned any word that represents group of 10 .


Figure 8. Materials for teaching number 10 Source: Developed by researcher

Especially, this item was asked to explore teachers' abilities about identifies methods and procedures afford instructionally. The findings suggested that very few respondents are used base-
ten blocks and discussed about base-10 structure while teaching number 10 .

### 4.3.3. Understanding students' misconception

The item was given about addition of two-digit number with carryover problem by giving student's solution as;

$$
\begin{array}{r}
26 \\
+35 \\
\hline 511
\end{array}
$$

Based on this problem, the question, 'What will you do to identify his misconception first?' was asked to identify strategy that teachers are used to understand students' misconception. The Figure 9 shows the result that only $4 \%$ respondents are used to ask question, $8 \%$ used to give similar problem for understanding students' misconception. Similarly, $83 \%$ respondents' explanation were like "I want to teach ...". Instead of using any strategies to understand students' misconception, they are seemed to be in a hurry to teach the content what they know rather than understanding the real problem of the student related to the misconception.


Figure 9. Strategy to understand misconception Source: developed by researcher

### 4.3.4. Anticipation of students' error

This item was asked with the purpose to find teachers knowledge on anticipate what students are likely to think and what they will find confusing or difficult or they might make error when completing a task. To fulfill this purpose, one word problem, "There are 13 books and each book has 62 pages. How many pages are altogether?" was given and asked to write possible error that students might do. The rubric (Table 5) for possible errors' were developed. Then

[^3]participant＇s responses were categorized into excellent，good，fair and poor based on number of error that they had mentioned as four errors，at least two and at most three errors，at least one error and no correct answer respectively．

Table 5：Rubric for students＇error

| Criteria | Descriptions |
| :--- | :--- |
| Reading \＆ <br> comprehension | recognize and decode the words or <br> symbols within the question（e．g．，not <br> understand problem，or confusion on <br> operation） |
| Transformation | choose an appropriate process or <br> algorithm（error to write mathematics <br> sentence） |
| Process skill | accurately do the operation <br> （multiplication or addition error） |
| Encoding | write answer to the question |

Source：developed by researcher based on Mukunthan （2013）

And，the Figure 10 shows the result of participant teachers about this item．


Figure 10．Responses about anticipation of error Source：developed by researcher

From analysis of Figure 10，researcher concluded that teachers＇knowledge is poor to anticipate what student will find confusing or difficulty to solve word problem of multiplication．

## 5．Areas for further study

Especially，the main purpose of the study is to explore the teachers＇CK and PCK in order to determine pre－conditions to develop students＇ mathematical thinking at grade 1－3 and apply new materials appropriately．The result of the study pointed out the baseline information of teachers＇CK and PCK to apply new curricular materials and revealed that the teachers＇CK and PCK are not
sufficient．A further study could explore teachers＇ support strategy to enhance teachers＇CK and PCK． Similarly，the study focused only on the content area of number sense and basic operation of grade 1－3 teachers＇CK and PCK．A further study could include more content areas of mathematics and other grade levels too．

## 6．Discussion and Conclusion

Deep understanding of mathematics knowledge is necessary but not sufficient to teach mathematics． And also it is not possible to teach effectively without having mathematical knowledge．So for effective teaching，teachers should be competent in both mathematical content knowledge and pedagogical content knowledge（An el at．，2004；Ball el at．，2008； Shulman，1986，1987；Gearhart and Saxe，2004）．

In contrast，the finding of the study shows that average level of respondents＇CK and PCK proficiency is below 40\％in excellent category．Else，around 33\％ teachers＇performance seems to be very poor in basic level skills of content knowledge．This result revealed that teachers＇mathematical knowledge for teaching is not sufficient．It clearly indicated one of the big issues related to insufficient performance of mathematics education in Nepal．

On the other hand，there is no significance difference between the teacher＇s mathematical knowledge between teacher who participated and did not participate on dissemination workshop about new materials．This result refers that content that discussed on workshop was not enough for teachers to enhance their knowledge about new change．From this disappointing finding，researcher wants to recommend special kind of teacher training should be given to the teachers in order to enhance their mathematical knowledge for teaching

Especially，base－10 structure is a platform for the children to develop their base－10 concept（Ong et al．， 2020）．Else，greater understanding of base－10 number structure should demonstrate greater use of base－10 decomposition strategy（Laski el at．，2014）．Even more， new curricular materials have strongly adapted the importance of base－10 structure（CDC，2019）and also it has been included to use base－10 decomposition strategy for addition and subtraction of single digit number．But teachers＇PCK on teaching number 10
shows that most of the teachers are used single digit counting and they did not discuss base-10 structure after counting while teaching number 10 . Moreover, the findings shows that most of the teachers are seemed to be confused in CK item about decomposition of numbers and also they have no knowledge about base-10 decomposition strategy for addition.

Furthermore, through analysis of respondents' response related to PCK item about the importance of problem solving and use of CPA in basic operation, it is found out that even though new materials prescribed to use problem solving and CPA sequence to teach basic operation, most of the teachers have no clear understanding about it. That shows the gap between teachers' practice and sequence of content of teaching that prescribed by new materials.

Similarly, another worrisome is that the finding revealed weak teacher's competencies on recognize and articulate students misconception. Also, teachers are not giving emphasis to understand reason of student misconception. Instead of that they are in hurry to teach the content what they know. Even more, teachers are seemed to be not confident to anticipate students' difficulties or error in specific task.

The findings of this study give the baseline information about teachers' knowledge for application of new curricular materials of mathematics at grade 1-3 properly. After analysis of these finding, researcher argues that teachers are not ready to apply new materials. They need professional development support about new changes and all those aspects or content those are more emphasized by new curricular materials. So it is suggested that professional development support for teacher is necessary to improve teachers' CK and PCK in order to apply new materials appropriately.

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[^0]:    ${ }^{1}$ For example, teacher's knowledge related to talk explicitly about how mathematical language is used, to choose, make and use mathematical representation effectively, and explain and justify one's mathematical ideas.

[^1]:    ${ }^{2}$ Number identification and Use of arithmetic signs are also seems to be challenging for the students（ERO，2020）

[^2]:    ${ }^{3}$ For $8+7$ ，think 8 and 2 make 10 ，separate 7 into 2 and 5 ，add 2 and 8 to make 10 ，add 5 more to get 15 （Laski et al．，2014）

[^3]:    ${ }^{4}$ count one by one ( $10=$ " $1,2,3,4,5,6,7,8,9,10$ ") (Laski et al., 2014)

