

Transformation of Mathematics Classes through Global Lesson Studies

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Abstract

In this study, based on the Global Lesson Study (GLS) program, a mathematics lesson study was conducted between teachers in Japan and Singapore, with Singapore as the host school. During the GLS process, the children's ideas—differed greatly between the two countries—were shared, and the lesson plans were improved to move toward integrated learning, which could not have been conceived during the in-school research discussions in Singapore, and then, the improvements were reflected in the lessons. This suggests that GLS is effective in improving the teaching of Mathematics.

Keywords: Global Lesson Study, Mathematics, Singapore, Japan

1. Introduction

With the spread and development of ICT in recent years, international exchange between teachers of different cultures is becoming easier. An international approach to conducting classroom research under the COVID-19 pandemic is an important issue and will provide important ideas for thinking about the future of global networks for lesson study after the pandemic. Good classroom practice requires the three elements of subject content knowledge, pedagogical knowledge, and technology knowledge, as well as Technological Pedagogical Content Knowledge (TPACK for short; Koehler & Mishra, 2009) as a link between these three elements, which can be improved through lesson study. It has been reported that teachers' qualities and abilities related to teaching are improved not only through domestic lesson study but also through global educational experiences such as Japan Overseas Cooperation Volunteers (JOCV) (Akai et al., 2019). In addition, using the VIDEO-LM framework of viewing, investigating, and discussing mathematics learning

environments, it was reported that Israeli teachers viewing Japanese mathematics lesson videos led to productive discussions among teachers (Schwartz & Karsenty, 2019). Also, as a case study of an intercultural collaborative lesson study project between Japanese and German universities, the following intercultural collaborative lesson study issues are identified: sharing data and culture, visualizing methodology and process, and responding to survey questions and answers. (Yoshida et al., 2021).

Based on these research trends, Sakai et al. (2021) defined the Global Lesson Study (GLS) as "international cooperative lesson study to understand different interpretations of teaching materials and teaching methods through international exchange among teachers using ICT, and to create lessons as a new value by integrating and developing them in education in each country" and developed a GLS program through a pilot global lesson study, which they defined as a Global Lesson Study (GLS). As an extension, the purpose of this study is to conduct a lesson study of Mathematics based on the GLS

program between teachers in Singapore and Japan, and to clarify the effect of GLS on the improvement of lessons.

2. Research Method

2.1. Participants of the GLS

As a partner of the GLS, we chose Singapore, which, like Japan, is a top-performing country in PISA and TIMSS, has introduced ICT and other technologies into education, and has a highly professional teaching staff. The teachers in Singapore are highly specialized Specialists because of the subject-teacher system. On the other hand, Japan is a generalist country because one teacher has to teach all subjects. Therefore, it is believed that Japanese teachers can improve their knowledge of subject content, pedagogy, and technology by interacting with Singaporean teachers through lesson study. On the other hand, for Singapore, which is trying to shift to collaborative learning with enriched language activities, Japan's practice of interactive and deep learning will be helpful and will lead to the improvement of pedagogical knowledge. As a result of this study, it is expected that teachers will be transformed by the improvement of TPACK.

Six public school teachers from Singapore who specialize in mathematics participated in the GLS. From Japan, five teachers from private university attached elementary schools participated, including one teacher specializing in mathematics.

2.2. Program of the GLS

The GLS has five stages: start-up, discussion before the research lesson, filming and observation of the research lesson, discussion after the research lesson, and closing. Based on the contents and methods of each stage of the GLS (Sakai et al., 2021), the GLS

was implemented in Singapore as the host school (H) and Japan as the guest school (G), following a program consisting of the following five stages.

- 1) Start-up: Selecting and proposing the grade level and unit for GLS implementation (H),
- 2) Discussions before the research lesson: Reviewing and proposing the study guide plan (H) → Making comment sheets (G) → Online meeting (H&G) → Revising the study guide plan (H),
- 3) Filming and observation of research lesson: Conduct and film the research lesson (H) → Upload video and notes (H) → Watch video and analyze lesson (G) → Create comment sheet (G),
- 4) Post lesson discussion: online meeting (H&G), summary of results and issues (H/G), and,
- 5) Closing: summary of modifications (H/G) → improvement of the program (H/G),

where "H&G" means both host school and guest school conduct the meeting together while "H/G" means host school and guest school work separately on the same task.

3. Overview and Discussion of the GLS

3.1. Startup

Based on the GLS program, the GLS was conducted from June to September 2019. As a startup, the host school proposed a unit on "Word Problems on Rate (Flow of Water)" to be studied in Grade 5 in Singapore, as shown in Figure 1. As shown in the English translation of the textbook (Keirin-kan; Shimizu et al., 2011) in Figure 2, this subject is also studied in Japan in the 6th grade unit on division of fractions. In the Singaporean curriculum, children in the fifth grade have not yet learned multiplication and division of fractions, but children in the sixth grade in Japan have already learned it, and it is expected

Example

Tap A alone can fill a tank to its brim in 8 minutes.

Tap B alone can fill the same tank to its brim in 12 minutes.

If both taps are turned on at the same time, how long will it take to fill the tank to its brim?

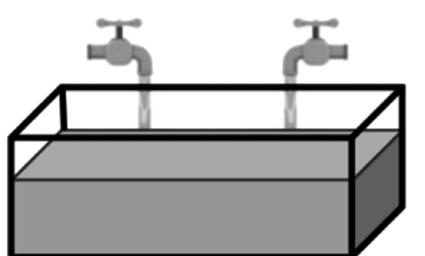


Figure 1. Word Problems on Rate (Flow of Water)

3 Two faucets can fill a tank with water.

Faucet A can fill the tank in 10 minutes, and
 Faucet B can fill the tank in 15 minutes.
 If both faucets are used at the same time, how many minutes will it take to fill the tank?

Figure 2. Treatment in Japanese textbooks (English version).

that there will be differences in the children's solution methods. Therefore, this subject was judged to be appropriate as a unit to be implemented in the GLS, as it would serve as an opportunity to "understand different interpretations of teaching materials and teaching methods, and to create classes as a new value by integrating and developing them in education in each country," which is the goal of the GLS.

3.2. Discussions before the Research Class

In preparation for the discussions prior to the research lesson, about five lesson plans were reviewed during the in-school research discussions at the host school in Singapore. In the discussion before the research lesson, the host school presented a lesson plan consisting of four questions as shown in Figure 3, where Problem 1 and 2 are questions to confirm previous learning, Problem 3 is the main problem, and Problem 4 is a problem aimed at utilizing and extending what was learned in Problem 3.

Problem 1 :
Water from a tap flows into an empty container at a rate of 400 ml per minute.
The capacity of the container is 12 litres.
How much time will it take to fill the container to the brim?

What does "capacity" mean?
 What does it mean "to fill the container to the brim"?
 What does "400ml per minute" mean?
 $1 \text{ litre} = \underline{\hspace{2cm}} \text{ ml}$
 $10 \text{ litres} = \underline{\hspace{2cm}} \text{ ml}$

Problem 2
Two taps can fill a tank with a capacity of 12 litres.
Water flows from Tap A at 600 ml per minute and from Tap B at 800 ml per minute.
If both taps are turned on at the same time, how long will it take to fill the tank to its brim?

What is the difference between Problem 1 and Problem 2?
 Will it take a shorter time or longer time for the tank to be filled to the brim? Why?

Problem 3
Tap A alone can fill a tank to its brim in 8 minutes.
Tap B alone can fill the same tank to its brim in 12 minutes.
If both taps are turned on at the same time, how long will it take to fill the tank to its brim?

Problem 4
An inlet tap can fill a tank in 5 minutes.
An outlet tap can drain the tank in 7 minutes.
Both taps are fully turned on at the same time. How long will it take to fill the tank completely?

Figure 3. Four problems to be presented in class.

In Japan, about two different problem situations are dealt with in one hour, but in this study plan, four problems are dealt with in one hour for the same the aquarium problem. In addition, it is not often seen in Japan that Problem 3, which is the central task of this lesson, and Problem 4, which is treated as homework (application problem), are in the same scene. Therefore, based on the host school's explanation that they took into account the difference in difficulty and structure of the problems from the concrete situations where the amount of water in the whole tank was presented in Problem 1 and 2 to the abstract situations where the percentage of the whole tank was used in Problem 3 and 4, the lesson development was shared. An understanding of the structure of lessons, from the "concrete" to the "abstract," from the "simple" to the "complex," and from the "manipulative" to the "formal," can be used to conceptualize the development of Universal Design lessons, and can lead to an improvement in subject content knowledge and pedagogical knowledge. For Problem 3, the host school presented the four expected solutions as shown in Figure 4. It was shared that Solution 1 is a typical misconception that we see in Japan. For Solution 2 and 3, it was explained that this is a solution method that is not seen in Japan, but is a typical solution method for 5th

grade students in Singapore who have not yet learned division of fractions, using the ideas of least common multiple and unit that they have already learned. As in Japan, the idea of unit was also found to be a major idea used in math education in Singapore. Regarding Solution 4, it was explained that in Japan, students use " $1 \div \frac{5}{24} = \frac{24}{5}$ " as a way of learning division of fractions, but in Singapore, students answer based on the meaning of fractions. In addition, the following scaffolding questions were set for each Solution.

[Scaffolding questions in Solution 1]

- How long does it take to fill the tank to the brim using Tap A alone?
- If Tap A and Tap B are used, will it take a longer or shorter time?
- Does your answer make sense?

[Scaffolding questions in Solution 2]

- Do you think you can draw a table to show the flow of water from Tap A and Tap B?
- What information do you think your table should contain?

[Scaffolding questions in Solution 3]

- Can you draw a diagram to show how much water from Tap A goes into the tank in 1 minute?
- Can you draw a diagram to show how much water from Tap B goes into the tank in 1 minute?

Solution 1

$$8 \text{ min} + 12 \text{ min} = 20 \text{ min} \text{ (wrong concept)}$$

Solution 2

$$24 \text{ min (from Tap A and Tap B)} \cdots 3 + 2 = 5 \text{ tanks}$$

$$5 \text{ tanks} \cdots 24 \text{ min}$$

$$1 \text{ tank} \cdots \frac{24}{5} \times 1 = \frac{24}{5} = 4\frac{4}{5} \text{ min}$$

| Tap A | | Tap B | |
|--------|---------|--------|---------|
| 8 min | 1 tank | 12 min | 1 tank |
| 16 min | 2 tanks | 24 min | 2 tanks |
| 24 min | 3 tanks | | |

Solution 4

$$1 \text{ min} \cdots \frac{1}{8} + \frac{1}{12} = \frac{3}{24} + \frac{2}{24} = \frac{5}{24}$$

$$5 \text{ units} \cdots 1 \text{ min}$$

$$24 \text{ units} \cdots \frac{1}{5} \times 24 = \frac{24}{5} = 4\frac{4}{5} \text{ min}$$

Solution 3

$$1 \text{ min} \cdots 3 \text{ units} + 2 \text{ units} = 5 \text{ units}$$

$$1 \text{ tank} = 24 \text{ units}$$

$$24 \div 5 = \frac{24}{5} = 4\frac{4}{5} \text{ min}$$

| Tap A | Tap B |
|-------|-------|
| 3 u | 2 u |
| 3 u | 2 u |
| 3 u | 2 u |
| 3 u | 2 u |
| 3 u | 2 u |
| 3 u | 2 u |
| 3 u | 2 u |
| 3 u | 2 u |
| 3 u | 2 u |

Figure 4. Four expected solutions.

- Is the number of equal parts the same in both diagrams? (No)
- Should the number of equal parts be the same? Why? (Yes, same tank)
- What do you have to do to make the number of equal parts the same in both diagrams? (Find common multiple)

[Scaffolding questions in Solution 4]

- What fraction of the tank is filled by water from Tap A in 1 minute?
- What fraction of the tank is filled by water from Tap B in 1 minute?
- What fraction of the tank is filled by water from both Tap A and Tap B in 1 minute?

As for these solutions, the host school presented the concept of obtaining 3 units and 2 units in Solution 3 and the concept of finding $1/8$ and $1/12$ in Solution 4 using the bar model as shown in Figure 5 on the iPad. This confirms, as in the case of Japan, the importance of explaining the relationship between the diagram and the equation. The understanding of Solutions 2, 3, and 4 and the understanding of scaffolding questions can be used to anticipate children's various ways of thinking and to provide individualized instruction, which will lead to the improvement of subject content knowledge and pedagogical knowledge.

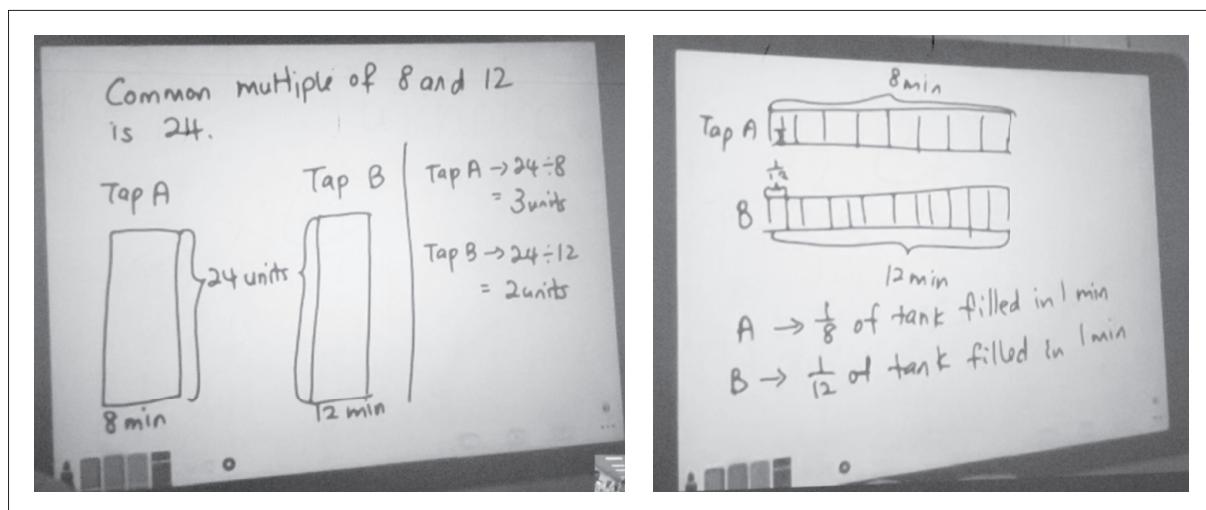


Figure 5. The concept of Solution 3 and Solution 4.

In addition, there was a discussion about the description of “integrating different ways of thinking as a key concept” in the lesson plan, and the host school shared the importance of “looking at the whole as a whole” and “using the least common multiple” as common points, and connecting them to a common key concept even if the solution methods are different. In this way, the importance of connecting them to a common key concept was shared. Regarding the gap between Problem 2, which is a concrete situation in which the total amount of water is presented, and Problem 3, which is an abstract situation in which proportions are used, the guest school showed the similarities in the way of thinking focusing on the unit amount as shown in Table 1, and suggested that the formula “1 tank divided by the amount of water that can enter in 1 minute” in Problem 2 should correspond to the respective Solution in Problem 3 as shown in

Table 2. By thinking integratively in this way, it was confirmed that the same formula could be formulated for Solution 4. Based on this, the host school proposed the idea that, as shown in Fig. 6, if we focus on the numerator for calculation $1 \div 5/24 = 24/24 \div 5/24$, we can arrive at the calculation $24 \div 5$, which is common to both Solution 3 and Solution 4.

Based on the above discussion, the lesson plan was improved into a learning process using Table 1 and Table 2 as a concrete way to integrate different ideas as key concepts. This kind of improvement in the lesson plans represents an improvement in subject content knowledge and pedagogical knowledge, as well as an “understanding of different interpretations of materials and teaching methods,” which is the goal of the GLS.

Table 1. Relationship between Problem 2 and Problem 3.

| | Problem 2 | Problem 3 |
|-------------------------|--------------------------|---|
| Capacity | 12 liters | 1 tank |
| Tap A (1 min) | 600ml (0.6 liters) | 8 min – 1 tank 1 min – 1/8 of tank |
| Tap B (1 min) | 800ml (0.8 liters) | 12 min – 1 tank 1 min – 1/12 of tank |
| Tap A and Tap B (1 min) | $0.6 + 0.8 = 1.4$ liters | ? ($1/8 + 1/12 = 5/24$ of tank) |
| Time | $12 \div 1.4 = 60/7$ min | ? ($1 \div 5/24 = 24/5$ min) |

Table 2. Relationship between Solution 2, Solution 3, and Solution 4.

| | Solution 2 | | Solution 3 | Solution 4 |
|--------------------------|-------------------------------------|-------------------------|--------------------|--|
| Capacity | 1 tank | Capacity | 24 units | 1 whole |
| Tap A (24 min) | 3 tanks | Tap A (1 min) | 3 units | 1/8 of tank |
| Tap B (24 min) | 2 tanks | Tap B (1 min) | 2 units | 1/12 of tank |
| Tap A and Tap B (24 min) | 5 tanks | Tap A and Tap B (1 min) | 5 units | 5/24 of tank |
| Time | $1 \div 5 = 1/5$ $1/5$ of 24 min | Time | $24 \div 5 = 24/5$ | $1 \div 5/24 = 24/5$ $(1 \div 5/24 = 24/24 \div 5/24 = 24 \div 5 = 24/5)$ |

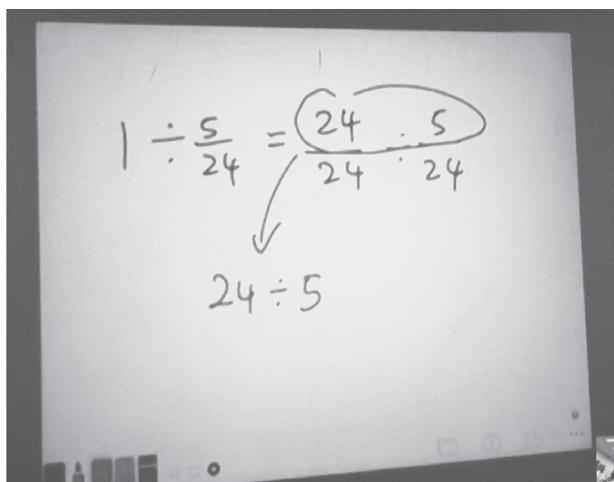


Figure 6. Illustration of the solution method using the idea of commutativity.

3.3. Discussions after the Research Class

Prior to the discussion after the research class, videos and photos of the research class and PDFs of the children's worksheets were uploaded to the cloud for the purpose of filming and observing the research class, and class observation and analysis were conducted by watching the videos.

In the discussion after the research class, it was explained that, as a result of this GLS, many children

were able to apply these ideas to different situations in Problem 4 by discussing and summarizing multiple solution methods so that they could easily notice the common key concepts in Problem 3, as shown in Figure 7. This is the result of the teaching practice based on the revision of the lesson plan, as shown in Fig. 8, in which ICT was used to connect the solutions in Problem 3 and Problem 4 with the typical Singapore method to enhance the strategy by having the students accurately grasp the method for the solution and utilize the method. The efficiency and effectiveness of this method was shared. This transformation of the lesson with the improvement of the lesson plan is a concrete example of the improvement of subject content knowledge and pedagogical knowledge, and the understanding of the use of ICT will also lead to the improvement of knowledge about technology. This lesson was created as a "new value that integrates and develops the understanding of different interpretations of teaching materials and teaching methods in the education of each country", which is exactly what the GLS aims for.

On the other hand, the guest school asked how the children who had the wrong idea as shown in

Figure 9 realized their mistake and corrected their idea. This is because in Japanese lesson study, there is a tendency to discuss whether the support was appropriate by analyzing the learning process of how the children with wrong ideas changed, rather than just the achievement of the goals as the outcome of the class. The host school explained that they had not been able to fully grasp the changes in the children's

thinking because the class was mainly focused on group learning activities. And therefore, the necessity of discussing how to provide support for the learning process, including the changes in the children, was confirmed. By focusing on this type of support, it is expected that the knowledge of subject content and pedagogical knowledge will be further improved.

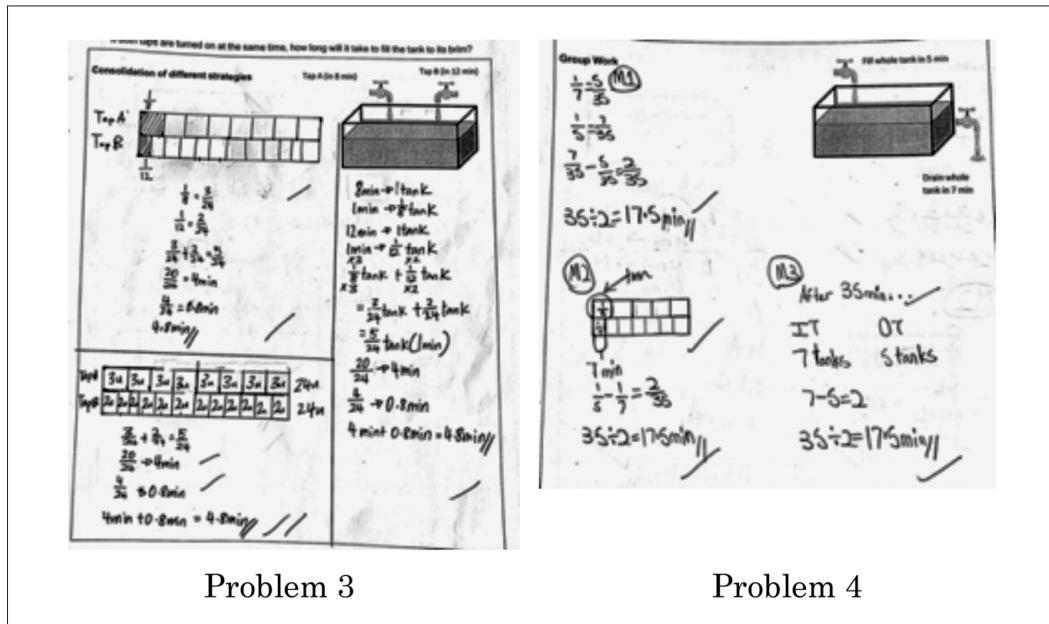


Figure 7. Notes for Problem 3 and Problem 4.

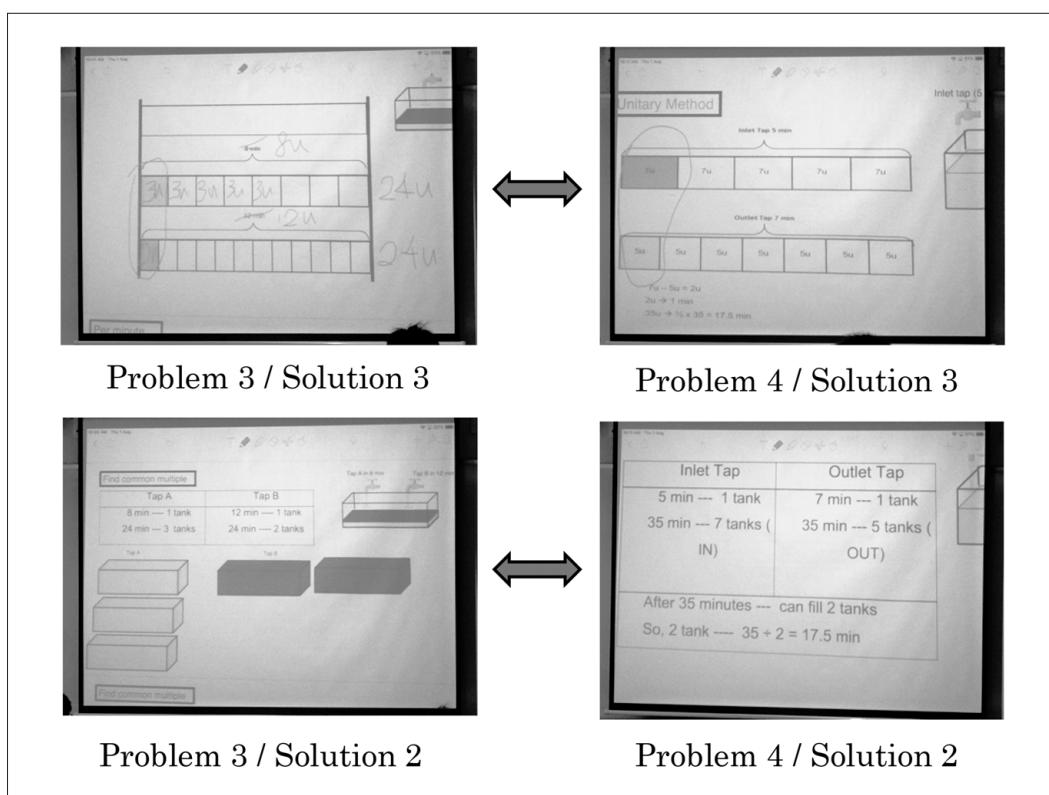


Figure 8. Correspondence between Problem 3 and Problem 4 as utilization of strategies.

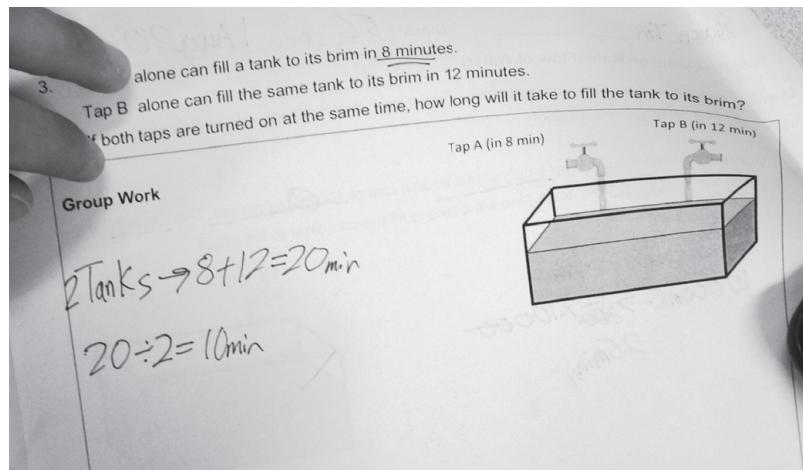


Figure 9. Children's wrong ideas.

3.4. Closing

The following comments on the results of the GLS were reported and shared via e-mail from the host and guest schools. From the comments, it can be read that both the host and guest schools felt that their subject content knowledge and pedagogical knowledge had been improved through the GLS.

<From the host school>

We were glad to find out that Singapore and Japan use similar type of questions in the teaching of Rate. The strategies taught were similar among the two countries. Our students mostly solved using the method of finding the tank filled per min.

However, after the discussion with Japan educators, we realised that the LCM method is easier for students to understand as they do not have to work with fractions, which is cognitively more demanding than working with whole numbers.

During that lesson, the teacher managed to pick up students' work from a few groups for class discussion but were not able to discuss other alternative solutions due to time constraint. Hence, in the next lesson, the teacher revisited the same word problem and used the LCM method to explain the question to students. The unitary explanation helped students to understand why 3 and 2 are chosen as they are the LCM of 24.

<From the guest school>

Thank you so much for such a big support for GLS. We almost complete this year activity. Especially thanks to you, we made it!! GLS in this year was also

fruitful time for us, indeed.

"Tank problem" is also familiar mathematical situation in Japan. Therefore, we were able to understand differences of knowledge already learned due to difference of curriculum and to consider differences in instruction through first Video conference. Since you showed us four solutions included misconception, we also learned students' solutions that was expected and different from math education context in Japan. Though time and quantity of water are different, dealing with the whole amount of water as "1" and using LCM are common methods/strategies in these problems. The Idea of integration regarding with this point is very important and required also in Japan. This implies and concludes your specialty in math was high and your interpretation of teaching materials was deep. GLS discussion was so interesting for us.

Among things we could achieve through GLS, one of the most fruitful outcomes was that through the discussion of the first meeting, lesson plan was well modified and contents of discussion were appropriately reflected to the lesson. I believe that this is one of the evidences that GLS would work for both country sides to improve lesson and foster teacher's viewpoints and skill to plan/do lesson.

On the other hand, we also learned through second Video conference. We understood how important the "Unit concept" is in mathematics education in Singapore and we were able to get some ideas to incorporate the "Unit concept", which is unfamiliar in Japan, into Japanese math classes.

4. Conclusion

In this study, based on the Global Lesson Study (GLS) program, an arithmetic lesson study was conducted between teachers in Japan and Singapore, with Singapore as the host school. During the GLS process, the children's ideas, which differed greatly between the two countries, were shared, and the instructional plan was improved to move toward integrated learning, which could not have been conceived during the in-school research discussions in Singapore, and the improvement was reflected in the teaching practice. Therefore, it can be concluded that the GLS is effective for improving lessons. It was also suggested that the GLS could lead to the improvement of the three components of subject content knowledge, pedagogical knowledge, and technology knowledge, and their connection to TPACK.

The GLS is a two-year cycle in which each country is in charge of a host school. Therefore, the future task is to implement the GLS with Japanese elementary schools as host schools and to verify the effectiveness of the two-year GLS.

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