# The Potential of the Global Lesson Study as PDCA+E: Verification through a Case Study in Mathematics 

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#### Abstract

In this study, based on the Global Lesson Study (GLS) program, an mathematics lesson study was conducted between teachers in Japan and Singapore, with Japan as the host school. In the process of the GLS, it was confirmed that the GLS functions as a Plan-Do-Check-Act (PDCA) cycle and that it is effective to implement the GLS in a two-year cycle when it is implemented among in-service teachers. As an "Extension", we proposed the following two points: (1) the possibility of action research by combining survey research and classroom practice, and (2) the possibility of a "GLS package" in the context of international educational cooperation.


Keywords: Global Lesson Study, PDCA Cycle, Extension, Mathematics

## 1. Introduction

With the rapid development of globalization against the background of an advanced information society, there is a need to develop teachers with intercultural competence in the field of education. Global educational experiences, such as overseas educational activities, are considered to be important for the development of such abilities. Under the COVID-19 pandemic, it is difficult to even go abroad, but with the spread and development of ICT in recent years, international exchange among teachers of different cultures is becoming easier. As one of the forms of international exchange among teachers, Sakai et. al. (2021) defined Global Lesson Study (GLS) as "international cooperative classroom research to understand different interpretations of teaching materials and teaching methods through international exchange among teachers using ICT, and to create
lessons as new values by integrating and developing them in education in each country". We are now developing the GLS program through the Pilot GLS in mathematics. Lesson study was born in Japan and has spread around the world as an effective method of teacher professional development (Fernandez, 2002; Isoda, 2007; Lewis et al., 1998; Stigler et. al., 1999,). As specific effects, there are studies that focus on the transformation of teachers' knowledge and beliefs (Lewis et al., 2009) and studies that reveal significant positive effects on increased teacher knowledge and skills and changes to instructional practice through lesson studies (Rock et al., 2005). Sakai et. al. (2021) proposed intercultural competence for lesson study by adding Outward Impact to the previous definition of intercultural competence (Deardroff, 2006), and proved that the GLS is not only effective in developing teachers with this competence, but also in improving teachers' competency in teaching subjects. In addition,

Sakai et al．（2022）conducted an mathematics lesson study between Japanese and Singaporean teachers based on the GLS program，using Singapore as the host school，and reported that the GLS was effective in improving mathematics lessons．

In addition，the structure of this lesson study is generally described as repeated cycles of planning， implementation，analysis，and revision（Fernandez， 2010；Lewis et al．，1998），and the process functions as a PDCA cycle（Eric，2018）．However，the GLS is a two－year cycle because the host and guest schools are changed every year．In the previous studies on GLS， the cycle itself was considered as a single project，and the fact that it functions as a PDCA cycle was not verified，nor were the possibilities as an Extension examined．

Therefore，the purpose of this study is to examine whether the GLS is functioning as a PDCA cycle in the process of the GLS，and also to examine what possibilities there are for the GLS as an Extension， and to make recommendations on how the GLS should be in the future．In addition，lesson study will be conducted to clarify the effect of GLS on lesson improvement．

## 2．Challenges and Improvements for the GLS in 2019

The two issues shared at the closing of the GLS in 2019 were as follows．
（1）Worksheets which we received were about ＂Consolidation of different strategies＂and these were not worksheets which were written in the class we observed．We needed worksheets of＂Group Work＂too，because we want to know how students＇ way of thinking changed or were improved．
（2）The recorded VIDEO which has captured around the front of the classroom was not so much． Therefore，we couldn＇t read and understand what was projected on the screen and how students learned during group activities．

Therefore，the following measures were considered in the 2020 GLS as remedies for these issues．
（1）Provide a copy of the notebook in which the children wrote their thoughts during the class，with the data translated into English．
（2）Prepare a camera to capture the whole class from the rear of the classroom and a camera for each
group to capture group activities，providing data for both the whole class and the groups．

## 3．Overview of the GLS

## 3．1．GLS Participants

Since Singapore was the host school for the GLS held from June to September 2019，Japan was the host school for the GLS in 2020．Six public school teachers specializing in mathematics from Singapore participated in the GLS，and five teachers from private university attached elementary schools participated from Japan．All were participating members of the GLS conducted in 2019.

## 3．2．Overview of the Startup

The GLS was conducted from July to October 2020 based on the GLS program，which consists of five stages：（1）startup，（2）discussion before the research lesson，（3）filming and observation of the research lesson，（4）discussion after the research lesson，and （5）closing．

As a startup，the host school proposed a unit on＂What is the original number？Going back one by one＂to be studied in the fourth grade in Japan，as shown in Figure 1．This subject is also treated as a three－element，two－step sentence problem in math education in Singapore，and is related to the problem－ solving strategy Working Backwards．In the case of Japan，the relationship diagram in Figure 1 is used， while in the case of Singapore，the bar model is used． This would lead to a better understanding of the solution method with different diagrams．Therefore， this subject was judged to be appropriate as a unit to be implemented in the GLS，as it would serve as an opportunity to＂understand different interpretations of teaching materials and teaching methods，and to create lessons as new values that are integrated and developed in education in each country＂，which is the goal of the GLS．


Figure 1. Word problems based on Reverse Thinking.

### 3.3. Summary of Discussions before the Research Lesson

Discussions prior to the research lesson focused on the problem-solving outlook, use of diagrams and solution methods, lesson development and problem structure, previously learned content, Reverse Thinking and Working Backward. The summary of the discussion before the research lesson is shown in Table 1. Regarding the use of diagrams and the solution method, in Japan, the focus is on understanding the relationship diagram, but the bar model in Singapore is easier for the children to understand, and other diagrams can be considered. The presence of a variety of diagrams leads to deep learning that focuses on the relationship between common quantities and relates different diagrams. Therefore, after the
discussion before the research lesson, the lesson plan was revised to include diagrams that show the relationship between quantities, instead of limiting it to relationship diagrams. This will lead to the GLS's goal of "understanding different interpretations of teaching materials and teaching methods, and creating lessons as new values that are integrated and developed in education in each country".

In addition, by focusing the discussion on a topic as shown in Table 1, it became easier to share and compare the learning content and teaching methods of each country in Japan and Singapore on the same topic. This allows us to focus on similarities and differences that are not limited to the units implemented in the GLS, and to develop an understanding of math education in each country, as well as its background.

Table 1. Summary of Pre-lesson conference

| Japan | Singapore |
| :---: | :---: |
| Organise "what is known" and "what is to be found" What is known: <br> - Bought 6 notebooks of the same price <br> - Bough a juice cost 100 yen. <br> - Total expenditure is 940 yen <br> What is to be found/calculated <br> - Price of a single notebook | We use Polya's 4 Steps in Problem Solving <br> - Understand the Problem <br> - Plan the Strategy <br> - Carry out the Plan <br> - Check <br> To help students understand the problem, we will also ask them to organise "what is known" and "what is to be found" |
| Strategy : Use Diagram | Strategy : Use Part-Whole Model (by Patrick) |
|  | Solution : <br> 6 notebooks $\rightarrow 940-100=840$ <br> 1 notebook $\rightarrow 840 \div 6=140$ |


| Japan | Singapore |
| :---: | :---: |
|  | In our school，we have a heuristic package in Primary 4 and one of the heuristic is the package is＂working backwards＂which is same as the concept of＂reverse thinking＂in Japan． <br> This is an example of a question in the package that involves＂working backwards＂． <br> Example <br> 20 passengers boarded the LRT at point B．When it arrived at point C， 4 passengers alighted and 12 passengers boarded it．At point D， 15 passengers alighted and 6 boarded the LRT．At point E，there were 40 passengers in all． <br> How many passengers were in the LRT at point A？ |
|  | Forward Thinking vs Reverse Thinking <br> Example 1 <br> There were 186 passengers on board a train．At a train station， 90 passengers got off the train and 25 passengers got on the train．How many passengers were there on board the train then？ <br> Example 2 <br> Alvin had some greetings cards at first．He used 98 greeting cards for Christmas．He bought another 452 greeting cards．Then he had 780 greeting cards．How many greeting cards did he have at first？ <br> Let students do the two questions and let them discuss what the difference in the strategy is． |
| Use problem 2 （ parallel problem with the same concept） as an application． <br> Problem 2： <br> You buy 8 apples．You got a 60 －yen discount and pay 900 yen．How much did each apple cost originally． | We practise I Do，We Do and You Do． <br> In I Do，teacher demonstrates how a problem is solved． In We Do，teacher and students solve a parallel problem together．Teacher helps to scaffold the solution by asking the students questions． <br> In You Do，students solve a parallel problem on his／her own． |
| Prior Knowledge of Reverse Thinking <br> Example in Grade 2 Math textbook ： <br> An apple cost 45 yen more than an orange．If an apple cost 100 yen，how much does an orange cost？ | Prior Knowledge of＂Working Backwards＂ <br> Primary 1 $\square+2=10$ <br> Will consider adding the heuristic of＂working backwards＂ in the Primary 2 and Primary 3 Heuristic Package． |

## 3．4．Filming and observation of the research lesson．

In Japan，instruction is mainly based on relational diagrams，but based on the content of the preliminary discussion，the lesson was modified to accept a variety of diagrams，such as bar models，and then implemented． In addition to filming the entire lesson with a fixed camera from the back of the classroom，we used iPads to film the students as they drew diagrams and solved problems during individual problem solving．For the video of the entire lesson，we recorded the audio interpreted from Japanese to English in real time．In consideration of COVID－19 infection，no activities such as group discussion were conducted in this research
lesson．After the class，eight videos of the entire lesson and of individual solutions were uploaded to the cloud， along with a copy of the notebook translated into English，in which the students wrote their thoughts during the class，as shown in Figure 2．In addition， data on the classification of children＇s thoughts as shown in Figure 3 was also provided as a resource to examine the change in children＇s thoughts．


Figure 2. Copy of a Child's Notebook.

| student | Main problem | Exercise | student | Main problem | Exercise |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | B | E | 21 | G | G |
| 2 | B | A | 22 | B | E |
| 3 | A | A | 23 | E | C |
| 4 | A | F | 24 | A | A |
| 5 | H | H | 25 | A | A |
| 6 | A | A | 26 | Absent | Absent |
| 7 | A | A | 27 | A | E |
| 8 | E | E | 28 | G | C |
| 9 | B | E | 29 | F | A |
| 10 | C | E | 30 | B | E |
| 11 | G | A | 31 | B | B |
| 12 | G | C | 32 | A | B |
| 13 | E | C | 33 | A | A |
| 14 | B | H | 34 | G | A |
| 15 | C | B | 35 | D | C |
| 16 | A | C | 36 | D | A |
| 17 | B | C | 37 | E | A |
| 18 | D | H | 38 | E | B |
| 19 | G | C | 39 | C | C |
| 20 | A | A | 40 | Absent | Absent |


|  | Categories |
| :--- | :--- |
| A | Line chart |
| B | Tape chart (bar model) |
| C | Relationship diagram |
| D | Real picture |
| E | Explanation and picture |
| F | Area diagram |
| G | Explanation and math sentence |
| H | No diagrams |


| Main problem | Frequency |
| :---: | ---: |
| A | 11 |
| B | 8 |
| C | 3 |
| D | 3 |
| E | 5 |
| F | 1 |
| G | 6 |
| H | 1 |


| Exercise | Frequency |
| :---: | :---: |
| A | 13 |
| B | 4 |
| C | 9 |
| D | 0 |
| E | 7 |
| F | 1 |
| G | 1 |
| H | 3 |



Figure 3. Classification data of children's ideas.

### 3.5. Summary of discussions after the research lesson

In the discussion after the research lesson, the focus was on the use of diagrams and interactive activities. Regarding the use of diagrams, since bar models are the main method in Singapore, no other diagrams are used. However, in this lesson, by allowing students to think using various diagrams, it was shown that students can solve problems using a variety of diagrams. From the results of the Main Problem shown in Fig. 3, it was shared that it is easy for students to think of solutions using line diagrams and tape diagrams (bar models). In addition, for the fact that some children changed the diagram used for problem solving in the Main Problem to a relational
diagram in solving the Exercise, the guest school asked whether the reason why they used the relational diagram even though the diagram was not specified was because they felt that the relational diagram was useful. This question gave Japanese teachers a perspective to deeply analyze their lessons. Thus, by sharing data for capturing children's changes, the project has evolved from a reflection on classroom practice as a lesson study to a practical research.

On the other hand, with regard to interactive activities, the guest school pointed out that although various diagrams were presented, there were few interactive activities between the teacher and the students, and that the students only took up various
ideas and summarized them on the blackboard．In this regard，the guest school introduced the following five practices from the teacher＇s standpoint as a way to get the students to discuss．
＂1．Anticipating＂：Anticipate how the children will respond，what ideas they will bring up，and how they will discuss them．
－Do the problem yourself．
－What are students likely to produce？
－Which problems will most likely be the most useful in addressing the mathematics？
＂2．Monitoring＂：Observe the child to understand how he or she is thinking．
－Listen，observe，identify key strategies．
－Keep track of approaches．
－Ask questions of students to get them back on track or to think more deeply．
＂3．Selecting＂：Since it is not possible to cover all the ideas，choose three，including not only the right ideas but also the wrong ones．
－CRUCIAL STEP－what do you want to highlight？
－Purposefully select those that will advance mathematical ideas．
＂4．Sequencing＂：Think about the connections between children＇s ideas，such as in what order they should be discussed．
－In what order do you want to present the student work samples？
－Do you want the most common？Present misconceptions first？
－How will students share their work？Draw on board？Use overhead camera（ OHC ）？
＂ 5 ．Connecting＂：The teacher facilitates the children to connect their own ideas．
－Craft questions to make the mathematics visible．
－Compare and contrast 2 or 3 students＇work －what are the mathematical relationships？ What do parts of student＇s work represent in the original problem？The solution？Work done in the past？
Looking back on this lesson from the perspective of these five practices，we found that the teachers were able to do well in＂1．Anticipating，＂＂2．Monitoring，＂ and＂3．Selecting＂．However，there was room for improvement in＂ 4 ．Sequencing＂and＂ 5 ．Connecting＂． The perspective of reflecting on this lesson can be
expected to be useful for the future development of lesson improvement and practical research through lesson study in Japan．

## 3．6．Overview of Closing

As a closing，the following comments were reported and shared from each of the host and guest schools on the GLS cycle and how to share GLS information．From the comments，it can be seen that both the host school and the guest school felt that the presentation of photocopies of the notebooks in which the children wrote their thoughts during the class and the materials to discuss the changes in the children＇s thoughts were useful for the discussions after the research lesson．The GLS cycle，in which the host and guest schools are rotated every year for a two－year cycle，was also effective in learning about math education in both countries．

## ＜From the host school＞

Based on reflections of last year，we prepared 2 kinds of video．One is video recorded a lesson by fixed video camera and the other is short video recorded focusing specific students．We also prepared copies of students＇ notebook where different and various diagrams were drawn．In addision we analyzed about transition of diagram which was drawn in students＇notebook．
Regarding this，short movie focused on specific students was effective in the meaning of reading students＇ideas．Since we focused instruction with diagram，copies of students＇notebook and analysis data of them were useful information for discussion．

## ＜From the guest school＞

Singapore has hosted the GLS project in the past three years．Our teachers have planned and conducted the lessons while our Japanese counterparts observe the lessons and give their comments．It is good that we switch the roles this year so that Singapore teachers have the opportunity to observe how a lesson is carried out in a Japanese classroom and how the lesson is being planned．
The data collected from the lesson such as photos of students＇work and analysis of the diagrams used is useful for discussion during post－observation conference．

## 4. Discussion

### 4.1. Functions as a PDCA cycle

The GLS program consists of the following five stages: PP (1) Start-up, P/D (2) Discussion before the research lesson, D/D (3) Filming and observation of the research lesson, CA/D (4) Discussion after the research lesson, and $\mathrm{A} / \mathrm{CA}$ (5) Closing, and is designed so that the first half of the two-year GLS cycle alone becomes a PDCA cycle. From the overview of each process of the GLS, it can be seen that even the latter half of the GLS is established as PDCA. In the second half of the GLS, Plan and Do are implemented based on the issues and improvement measures in the first half, and it is thought that the quality of the GLS has been improved by the spiral PDCA cycle in the twoyear cycle of the GLS. Therefore, the GLS functions as a PDCA cycle and it is considered effective to implement it in a two-year cycle.

## PD (PDCA) CAE

The GLS program consists of five stages centered on lesson study: (1) start-up, (2) discussion before the research lesson, (3) filming and observation of the research lesson, (4) discussion after the research lesson, and (5) closing. These will be examined from the perspective of PDCA.

First, in the core activity of lesson study, lesson plans were presented by the host country in "discussion before the research lesson (step 2)". The lesson plans are then discussed between the host and guest teachers with the background of each country's pedagogy, and the final lesson plans are prepared. From these points of view, setting the theme in step 1 and step 2 are positioned as "Plan" in lesson study. Next, the lesson plan planned in step 2 is put into practice in step 3, which is "Do" in class research. Step 4 is the Check and Action section, which analyzes (Check) the lessons implemented (Do) based on the lesson plans (Plan) and discusses how to further improve (Action) the instruction.

However, the GLS program itself can also be interpreted using the PDCA cycle. Step 1, where the GLS program is presented, is the Plan. Then, "Do" is the part related to the implementation of class research, which is linked to Steps 2-4. Step 5, which is the review of the program itself, can be positioned as "Check" and "Action".

In other words, as shown in Figure 4, the GLS program itself can be positioned as a PDCA cycle (bold in the figure) in a way that the activities of PDCA (italics in the figure) in lesson study are contained within it.


Figure 4. Structure of GLS and LS as PDCA

In this second year of the GLS, there were some major changes from the first year. As shown in the outline, the recording method for the class research was from the back of the classroom in the first year, whereas in the second year, in addition to filming from the back of the classroom, the children's activities were filmed with multiple iPads. It was also in the second year that copies of the children's notebooks and information on the classification of the children's thinking were provided as materials for the postlesson discussion. These reflect the feedback from the first year guest country (Japan) as shown below at the
closing of the first year of the GLS.
(1) Video included much noise and howling, so it was hard to hear and understand voices of teacher and student.
(2) Worksheets which we received were about "Consolidation of different strategies" and these were not those which we observed in the lesson on August 1st. We needed worksheets of "Group Work".
(3) The recorded VIDEO which has captured around the front of the classroom was not so much. Therefore, we couldn't read and understand what
was projected on the screen．
These three points were pointed out at the closing as factors that prevented the first year＇s post－class review meeting from being sufficiently deepened，and are positioned as＂checks＂and＂actions＂in the GLS program．

The quality of the program was improved in the second year based on the＂Check＂and＂Action＂of the first year，as shown in the following statement：＂By sharing data for capturing children＇s changes，the program has developed from a reflection on lesson practice as a lesson study to a practical research．＂This impact on the quality of the program，which is not limited to the creation of lessons in the lesson study， can also be read from the descriptions of the teachers on the guest country（Singapore）side in the second year．The first part，where it says＂it is good that we switch the roles this year so that Singapore teachers have the opportunity to observe how a lesson is carried out in a Japanese classroom and how the lesson is being planned，＂is a description of lesson creation and practice，and can be considered a reflection on classroom research．On the other hand，the latter part， where it says＂the data collected from the lesson such as photos of students＇work and analysis of the diagrams used is useful for discussion during post－observation conference，＂is not a suggestion about pedagogy，but a reference to a system of class research，namely the GLS program．In addition to the improvement of the participants＇pedagogy－related qualities and intercultural competence through the PDCA cycle of class research，Figure 5 shows that the GLS program itself functions as a PDCA cycle and improves its quality through the participants themselves．


Figure 5．PDCA spiral in the GLS．

And this improvement in the quality of the GLS program will lead to an improvement in the quality of lesson study，which will lead to more effective improvement in the participants＇qualitative abilities． In this way，they are not two separate and independent things，but rather they are in a relationship of mutual influence．

This indicates that the quality of the GLS as a program can be expected to improve through the PDCA cycle by making it a two－year reciprocal program rather than a single－year class study．

## 4．2．Possibility as Extension

## （1）Development as action research

By sharing and comparing the learning contents and teaching methods of Japan and Singapore on the same topic in the discussion before the research lesson， the teachers were able to focus on the similarities and differences that are not limited to the unit implemented in the GLS，and developed their understanding of math education in their respective countries and its background．With regard to the change in recording methods from the first year to the second year mentioned above，participants are also thinking about what kind of materials and information would be needed to facilitate more post－lesson discussions．This process was a good opportunity to think about how to observe and evaluate children＇s learning and behavior in class practice．In this process，we shared data to capture children＇s changes in the discussions after the research lesson，and by reflecting on children＇s changes and the reasons for them，we developed from ＂practical lesson study＂to＂practical research＂to analyze children＇s thinking processes．Furthermore， by reflecting analytically on class practices from the perspective of the five practices as a means of getting students to discuss，improvements related to ＂4．Sequencing＂and＂ 5 ．Connecting＂are suggested．In other words，by applying the theory of＂ 5 －practice＂to the interpretation of practice，issues are discovered and further＂Action research＂is developed to resolve these issues．

Both of these points relate to the analytical perspective of classroom observation．The former is a proposal from the Japanese side to the Singaporean side，and the latter is a proposal from the Singaporean side to the Japanese side．When conducting class research between two countries with different
educational backgrounds, the perspectives and viewpoints from which to observe and analyze the lessons were also different. In order for these two countries to discuss the same lesson, it is necessary for them to view the lesson from the same point of view and perspective as they provide and share their perspectives. These activities are expected to lead to a more analytical interpretation and discussion of the lessons.

In this way, the GLS has the potential to develop as Action Research through a back-and-forth between research and classroom practice.

## (2) Development as the GLS package

The GLS for 2019 and 2020 was conducted based on the GLS program under the condition that the participating countries were both advanced education countries, Singapore and Japan, and the participants were in-service elementary school teachers. The GLS program can be implemented anywhere there is an ICT environment.

The GLS was started as an "international cooperative class research project to understand different interpretations of teaching materials and teaching methods through international exchange among teachers using ICT, and to create classes of new value by integrating and developing them in the education of each country." However, it is important that the collaboration be cross-cultural, and not limited to countries with advanced education. The GLS between developed and developing countries provides an opportunity for developing countries to learn about the best teaching theories and methods of developed countries. On the other hand, for teachers in educationally advanced countries as well, the activity of explaining to teachers in the other country about teaching methods and practices that are taken for granted in their daily environment provides an opportunity to question the meaning and value of such methods and practices, leading to essential and deeper understanding.

In addition, participants are not limited to inservice teachers. In addition to in-service teachers, participants may include university faculties, students in teacher training programs, graduate students, educational consultants, and many others. This diverse combination of "participating countries \& participants" suggests that the know-how of GLS
could be developed into a "GLS package" based on the GLS program, even in the context of international educational cooperation.

## 5. Conclusion

In this study, the GLS functioned as a PDCA cycle in the process of the two-year cycle, and it was confirmed that it is effective to implement the GLS in a two-year cycle when it is implemented among inservice teachers. In addition, we propose the following two points regarding the possibility of the GLS as an "Extension".
(1) Possibility of Action Research by combining research and classroom practice
(2) Potential as a "GLS package" in the context of international educational cooperation
The future task is to establish a "GLS package" by implementing the GLS as part of the follow-up of those who have studied in Japan as a short or long term teacher trainee in order to verify the potential of the GLS in international educational cooperation.

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